Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. Donnell, *Editor* T. von Kármán, S. Timoshenko, *Editorial Advisers*

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Abbreviations of units follow the standard of Abbreviations for Scientific and Engineering Terms of the Am. Standards Assoc. Examples: psi (pounds per square inch); cps (cycles per second); mph (miles per hour).

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

March 1948 Vol. 1, No. 3

A Message About the London Congress From Sir Richard Southwell

THE Seventh International Congress of Applied Mechanics will be held at the Imperial College of Science and Technology, South Kensington, London, England, Sept. 5 to 11, 1948. Notices have been sent to participants at former Congresses, to Universities, to learned societies, and to technical journals. Application forms can be obtained by writing to the Organizing Secretary.

Previous meetings of the Congress were held at Delft (1924), Zurich (1926), Stockholm (1930), Cambridge, England (1934), Cambridge, Mass. (1938), Paris (1946). I believe that the decision to hold the Delft Congress was made at a more informal meeting at Innsbruck in 1922, but my own memory goes no further back than 1924. The pattern adopted for the Delft Congress, whose organizers were Professors C. B. Biezeno and J. M. Burgers, has been maintained ever since. Starting on Sunday and terminating on the following Friday, technical sessions are organized under the following sections: (I) Elasticity and Plasticity; (II) Aerodynamics, Hydrodynamics, Meteorology; (III) Thermodynamics, Heat Transfer, etc.; (IV) Vibrations, Lubrication, Numerical and Experimental Methods. In addition lectures or surveys are delivered on topics of general interest.

Deep gratitude is due to our French colleagues who, despite all difficulties, arranged a highly successful Sixth Congress, in the Sorbonne, little more than a year after the cessation of war in Europe. At that Congress the International Committee considered the question of the year and site of the next Congress. It appeared that a meeting in 1948 (instead of 1950) would be desirable, to avoid clashing with an International Congress for Pure Mathematics. Since there was some question about finding a country to act as hosts within so short a time, I offered (very tentatively, since the Congress had already honored England with a visit) hospitality in September 1948 at the Imperial College of which I am the Rector. In the autumn of the same year I learned, to my great pleasure, that this offer had been accepted.

These Congresses from the first have been informal gatherings of "working scientists"—men actually engaged in original research. They are not concerned with the "organization" of scientific work, but with the work itself. To many, I think, their informality is their chief attraction, and we who have been entrusted with the reception of this Seventh Congress have tried to maintain that feature. If our organization is criticized, as we know it must be, the charge may be that it is too informal, but we believe that what most members seek is an opportunity to meet their colleagues in other countries, and to share their problems and compare their experiences.

Arising out of the Paris Congress, there has been created a Union of Theoretical and Applied Mechanics, and on questions of major principle we work under the guidance of its Council. This Council has decided to invite Italian scientists to the 1948 meeting, and to leave for discussion at that time the question of inviting German scientists to later Congresses. Our invitations have been issued accordingly.

It was not expected that organization would be easy, and I dare not hope that all our difficulties will have been surmounted when we greet our visitors next September. They will, I am sure, be lenient to shortcomings. The greatest difficulties—due, of course, to present circumstances—come not from *scarcity* but from *uncertainty* of what may be permitted. Most of the friends from overseas whom I have met in the last two years seem to have found conditions of life in England less austere than they had expected, and I do not think that our visitors need fear that in South Kensington they will suffer grave discomfort. But scarcities result in regulations, and regulations are difficult to forecast. My efforts to arrange the traditional Congress Dinner have centered not on insuring that a dinner will be there to eat, but on obtaining release from a restriction (now withdrawn) of the number that may participate.

We are even now unable to say with certainty whether it will be possible to print Proceedings of the Congress. To cover expenses we must be assured of an adequate demand before we start to print. This can only be definitely ascertained by having members give undertakings to subscribe, at the time of the Congress, and for this a definite price must be quoted. We have instituted inquiries, and I hope that when the Congress meets we shall be able to state a price. Much depends on the allocation of paper, and here again it is unsafe to predict. English printers seem unwilling to print papers except in English, and on that account we have felt bound to notify authors that, while French is of course an accepted language of the Congress, translation into English may be an essential condition of a paper's being printed.

Enrollments received to date are satisfactory and give promise of a very successful meeting. We are doing all we can to insure success, for we are very conscious of the honor done our College by its choice as the place of meeting. Where numbers at gatherings have to be restricted, we will issue tickets in accordance with the order of enrollment; but we hope to welcome all our guests at the initial sherry party, held between 4:30 and 7:00 p.m. on Sunday, Sept. 5, 1948.

R. V. Southwell

General Theoretical and Experimental Methods

(See also Revs. 395, 410, 452, 460, 494, 495, 543)

390. D. B. Topolianski, "On approximations to Dirichlet's integral" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Sept.-Oct. 1947, vol. 11, pp. 551–554.

This paper describes an approximation *from below* to the Dirichlet integral. Michael Sadowsky, USA

General Dynamics, Kinematics, Friction

(See also Rev. 394)

391. N. Rozovsky, "Plans of moments and plans of moment ratios of regular toothed gear mechanisms" (in English), *C. R. Acad. Sci. URSS*, Mar. 30, 1947, vol. 55, no. 9, pp. 797–799.

The author devises certain geometrical constructions to facilitate the establishment of the relations between the moments applied externally to the basic links of a regular toothed gear mechanism, which produces either two or three independent relations between the angular velocities of its basic links.

Ian N. Sneddon, Scotland

Gyroscopics, Governors, Servomechanisms

392. F. Galavics, "The discontinuous regulator with auxiliary force and its relation to the continuous regulator (Der unstetige Regler mit Hilfskraft und seine Zurückführung auf den stetigen Regle)," *Schweiz. Arch.*, Dec. 1947, vol. 13, pp. 357–370.

This article discusses the problem of a regulator controlling the level of a fluid by means of a servomotor with electrical relays. The speed of the motor is unrelated to fluid quantities. In addition, the article treats the general subject of continuous and discontinuous regulators.

Mechanical simplification is usually possible when using a discontinuous regulator of the type mentioned above, but certain operating difficulties arise. Analysis by linear differential equations in the classical manner is not possible as with continuous regulators, but a step-by-step solution is not usually difficult to make, and the outline of such a solution is presented in this article. The discussion is centered on the problem of stability.

Several novel ideas are included, and the treatment is very different from that based on trajectories in the phase plane, though somewhat less general. No reference is made to articles in English on continuous or discontinuous (relay-type) servomechanisms. W. C. Johnson, Jr., USA

Regarding Publication Delays

THE editors and publishers of APPLIED MECHANICS REVIEWS wish to ask their readers' forbearance for publication delays. Many American journals have been held up in recent months due to the current situation in the printing industry, and there are additional factors which have delayed AMR. Besides the difficulties in getting a new venture started, the printing and mailing situation is still chaotic in many European countries (one Italian publication has just brought out a 1945 issue!) and this seriously affects a review journal which depends on such sources. If our first issue had been dated "March" these delays would have been less apparent, but it was decided to date it "January" and to gradually catch up with the calendar later, in order to simplify the relation between the volumes and the years of our journal throughout its future life.

Vibrations, Balancing

(See also Rev. 543)

393. Adam T. Zahorski, "Free vibrations of sweptback wing," *J. aero. Sci.*, Dec. 1947, vol. 14, pp. 683–692.

As a basis for this analysis, the symmetric and antisymmetric modes of vibration and natural frequencies are computed for an idealized sweptback wing having a uniform cross section. This solution is obtained from the differential equation governing the motion of a uniform beam subjected to bending and twist. The Rayleigh-Ritz method is then used to obtain the solution for the nonuniform wing, where the terms in the series are made up of the configurations of the uniform sweptback wing. Five elastic modes of vibration are considered. Formulas are given for the uniform wing and typical results are given for a nonuniform wing.

The enormous labor involved in getting an answer for a practical case, and the assumptions that must be made to handle the problem at all, make it seem likely that equally practicable results may be more readily obtained by an entirely numerical procedure, such as the iteration procedure, the Myklestad procedure, or any of the variations of these. N. M. Newmark, USA

394. C. B. de Carbon, "Improvement in the suspension of road vehicles (Perfectionnement à la suspension des véhicules routiers. Amortisseur à relaxation)," *C. R. Acad. Sci., Paris*, Oct. 27, 1947, vol. 225, pp. 722–724.

Whereas the classical shock absorber consists of a spring and hydraulic damper in parallel, the author suggests placing a further spring in series with the hydraulic damper, calling this arrangement a relaxed shock absorber. Improved riding qualities are claimed in practice and proved theoretically. In particular, for the same static stiffness a higher damping coefficient can be achieved with a smaller hydraulic unit.

P. C. Dunne, England

395. D. I. Sherman, "On the Dirichlet and Neuman problems in the theory of steady oscillations" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Mar.-Apr. 1947, vol. 11, pp. 259–266.

This is a mathematical proof of the existence of a solution of the partial differential equations related to vibrations of two-dimensional (plane) bodies. Michael Sadowsky, USA

396. S. J. Loring, "Experimental determination of vibration characteristics of structures," *Proc. Amer. Soc. civ. Engrs.*, Dec. 1947, vol. 73, pp. 1457–1474.

A simple instrument for determining vibration characteristics of structures is described. The device consists of a frame suspending a mass between two linear springs. Both mass and spring can be varied to obtain a wide range of natural frequencies.

When the frame is attached to a structure, the mass normally vibrates as if the frame is attached to a rigid support. However,

if the test mass frequency is tuned to a natural frequency of the structure the *rate of decay* is modified.

Two cases occur. In the first case the amplitude decays in an oscillatory manner, and observations of the time between two successive peaks and zeros give immediately the damping coefficient and the effective mass of the structure in the given mode. In the second case the amplitude decays exponentially and the rate is greatest at the tuning point. By observing the rate of decay at a number of frequencies about the tuning point, it is again possible to determine the vibration characteristics of the structure.

J. Hadji-Argyris, England

397. G. Horvay and S. W. Yuan, "Stability of rotor blade flapping motion when the hinges are tilted. Generalization of the 'rectangular ripple' method of solution," *J. aero. Sci.*, Oct. 1947, vol. 14, pp. 583–593.

The authors apply an approximate method of solution of certain differential equations (the so-called ripple method) to the problem of determining the flapping transient of a helicopter rotor blade. The results obtained are interesting, but their importance suffers from the restrictions imposed by the simplifying assumptions.

N. O. Myklestad, USA

398. Jacques Valensi, "Oscillations of a heavy viscous liquid in a U-tube of small diameter—I. Theoretical study (Oscillations d'un liquide pesant et visqueux dans un tube en U de faible diamètre—I. Étude théorique)," *C. R. Acad. Sci., Paris*, Feb. 17, 1947, vol. 224, pp. 446–448.

In this note the author shows that the significant parameter of the oscillatory motion of a heavy viscous fluid in a U-tube of small diameter is $\frac{R^2\omega_p}{\nu}$, where R is the radius of the circular cross section of the tube, ω_p is the frequency of oscillation of the same column of inviscid fluid, and ν is the kinematic viscosity.

A theoretical criterion is proposed for the transition from a laminar to a turbulent form of oscillatory motion. The basis of this criterion is the vanishing of the instantaneous velocity profile gradient at the wall of the tube.

R. C. F. Bartels, USA

399. Jacques Valensi and Mlle. Claire Clarion, "Oscillations of a heavy viscous liquid in a U-tube of small diameter—II. Experimental verification (Oscillations d'un liquide pesant et visqueux dans un tube en U de faible diamètre—II. Vérification expérimentale)," *C. R. Acad. Sci., Paris*, Feb. 24, 1947, vol. 224, pp. 532–534.

The experimental data obtained earlier by M. Menneret ["Oscillatory motion and uniform motion of a liquid in cylindrical tubes—Coefficient of internal friction," Imprimerie Allier Frères, Grenoble, 1911] are reinterpreted for the purpose of verifying the results of the analytical treatment of the oscillatory motion of a heavy viscous fluid in a U-tube by one of the authors in the paper reviewed above.

R. C. F. Bartels, USA

400. Jacques Valensi and T. Vogel, "The oscillation of a heavy viscous liquid in a U-tube (Sur l'oscillation d'un liquide pesant et visqueux dans un tube en U)," *C. R. Acad. Sci., Paris*, June 16, 1947, vol. 224, pp. 1695–1697.

Pérès and Goldstein have observed that the solution obtained by Valensi [*C. R. Acad. Sci., Paris*, Feb. 17, 1947, vol. 224, p. 446] is incomplete. The present authors extend his study. A general solution of the equation of motion satisfying the limiting conditions is found in the form of a series involving a complete set of orthogonal functions. The similarity parameter governing the phenomenon is $R^2 (\omega_p/\nu)$, where R is the radius of the tube, ω_p the period of oscillation which would be obtained with an inviscous fluid, and ν the kinematic viscosity.

The behavior of the solution, depending on the value of this parameter in relation to the zeros of the zeroth order Bessel function, is discussed.

Stuart R. Brinkley, Jr., USA

401. S. Simpkinson, L. J. Eatherton, and M. B. Millenson, "Effect of centrifugal force on the elastic curve of a vibrating cantilever beam," *Nat. adv. Comm. Aero. Tech. Note No. 1204*, Feb.1947, pp.1–23.

A study was made to determine the effect of rotation on the dynamic stress distribution in vibrating cantilever beams. The results of mathematical analyses are presented, together with experimental results obtained by means of stroboscopic photographs and strain gages.

It was concluded that high vibratory stress positions are unaffected by the addition of centrifugal force at rotary speeds as high as 100 per cent above the normal operating speed range of present aeronautical equipment. Nonrotating vibration surveys of blades therefore are valuable in predicting high vibratory stress locations under operating conditions.

N. O. Myklestad, USA

Wave Motion, Impact, Seismology

(See also Revs. 408, 497, 563)

402. F. Ursell, "The effect of a fixed vertical barrier on surface waves in deep water," *Proc. Camb. phil. Soc.*, July 1947, vol. 43, pp. 374–382.

The two-dimensional reflection and transmission of a train of waves advancing from infinity toward a vertical barrier in deep water is considered for various positions of the barrier. Two different expressions are obtained for the velocity potential on either side of the barrier, and the condition that the velocity of the fluid must be continuous in the plane of the barrier (except across it) then leads to an integral equation which is solved analytically.

The two cases where the barrier either extends from some point above the surface downwards, or from the bottom upwards, are considered in detail. For these cases, the reflection and transmission coefficients are expressed in terms of cylindrical functions (Bessel functions and associated functions).

Some numerical results are given and reference is made to experimental work on the subject. Conditions arising from other positions of the barrier are considered in outline.

A. Robinson, England

403. W. R. Dean, "Notes on waves on the surface of running water," *Proc. Camb. phil. Soc.*, Jan. 1947, vol. 43, pp. 96–99.

This paper deals with the waves produced by the application of pressure to the surface of a stream. The problem is taken as two-dimensional, and it is assumed that the pressure is applied over a band of finite width. The same problem was considered by Rayleigh, Lamb ["Hydrodynamics," Cambridge Univ. Press, 1932, art. 242] and others.

In order to make the problem determinate these previous authors assumed that there is a frictional force, however small, acting in the fluid and proportional to the relative velocity of

waves and free stream. In the present paper, this is replaced by the assumption that the disturbance vanishes at points far upstream from the region of application of the external pressure. It is shown that far downstream the result of the analysis is in agreement with that obtained by Lamb.

A. Robinson, England

Acoustics

404. C. W. Hicks and H. H. Hubbard, "Comparison of sound emission from two-blade, four-blade, and seven-blade propellers," *Nat. adv. Comm. Aero. Tech. Note No. 1354*, July 1947, pp. 1–34.

Measurements of sound pressures are presented for the two-blade, four-blade, and seven-blade propellers in the tip Mach number range 0.3 to 0.9. For two-blade propellers the experimental results were found to check satisfactorily with those calculated over the whole Mach number range tested. In the case of the seven-blade propeller agreement was good for Mach numbers above 0.5, but large discrepancies existed in the range below 0.5.

A. D. Kafadar, USA

405. J. W. Miles, "The diffraction of sound due to right-angled joints in rectangular tubes," *J. acoust. Soc. Amer.*, July 1947, vol. 19, pp. 572–579.

This article presents a method of analyzing the diffraction of sound due to right-angle joints in rectangular tubes, by setting up an electrical analogy. The general form of the network derived is that of a lattice network and it can be handled by conventional methods. An example of the application of the end results is presented.

The derivations are not easy to follow, due partly to the author's use of superscripts as indices. The stated analogy of voltage for velocity and current for pressure appears to be a proofreader's error. Reference to the author's two previous papers on similar subjects might assist a reader in following the derivations.

R. G. Wilson, USA

406. R. H. Nichols, Jr., H. P. Sleeper, Jr., R. L. Wallace, Jr., and H. L. Ericson, "Acoustical materials and acoustical treatments for aircraft," *J. acoust. Soc. Amer.*, May 1947, vol. 19, pp. 428–443.

Soundproofing methods and materials for reducing noise in the interior of aircraft are treated comprehensively in this article. Desirable qualities and arrangements of materials are discussed and numerous diagrams, tables, and curves are presented. Little theory is given, it being unnecessary in the treatment of the subject.

In particular, limitations to what can be done in this field are brought out. It is shown that acoustic treatment alone will never be adequate to reduce to a satisfactory level engine and other external noises. The quantitative data presented might very well serve as handbook material for aircraft designers to use for acoustic treatment of airplane fuselages.

R. G. Wilson, USA

407. E. Y. Yudin, "On the vortex sound from rotating rods," *Nat. adv. Comm. Aero. Tech. Memo. No. 1136*, Mar. 1947, pp. 1–10 [trans. from *J. tech. Phys. (Zh. tekh. Fiz.)*, 1944, vol. 14, p. 561].

The author treats in an approximate manner the acoustic power radiated from rotating rods and blades, and its dependence on tip speed and aerodynamic properties of the rotating bodies. The radiated vortex sound is shown to be proportional to the product of the sixth power of the tip speed, the effective area of the blade, the second power (however, in a footnote the translator, E. Z. Stowell, states that in his opinion this should be the fourth power) of the form-drag coefficient, and the square of the Strouhal number.

Some experimental results were obtained which are in general qualitative agreement with the theoretical predictions.

Albert London, USA

408. Carl Eckart, "Vortices and streams caused by sound waves," *Phys. Rev.*, Jan. 1, 1948, vol. 73, pp. 68–76.

This article presents a most interesting treatment of the lateral flow caused by the presence of sound fields in fluids. Examples of this phenomenon are the apparent flow of air from a Helmholtz resonator, and the violent air currents near vibrating quartz and magnetostrictive oscillators.

The history of the study of this phenomenon is discussed in some detail.

The flow is shown to be due to vorticity produced as a second order effect in intense sound fields. The derivation of the theory is quite straightforward and clearly presented. The results of the theory are shown to be in good agreement with experimental results.

R. G. Wilson, USA

Elasticity Theory

(See also Revs. 419, 420, 423, 424, 427)

409. C. J. Tranter and J. W. Craggs, "Stresses near the end of a long cylindrical shaft under nonuniform pressure loading," *Phil. Mag. London*, Mar. 1947, vol. 38, pp. 214–225.

The authors investigate the stresses in a semi-infinite solid cylindrical shaft due to a uniform pressure applied over a finite band at the end of the shaft. The problem arises technically when a ring is shrunk on or near the end of a shaft The analysis is based on a method due to F. Purser for the treatment of stress distributions in cylinders of finite length in which the boundary conditions can be represented by Fourier series.

John Hadji-Argyris, England

410. Sir Charles Inglis, "Analytical determination of shear stresses and torsion stresses in beams and shafts of any given uniform section," *J. Instn. civ. Engrs.*, Nov. 1947, vol. 29, pp. 19–62.

The problem of determining the shearing stresses in a uniform bar, under St. Venant bending or torsion, is first reduced in a standard manner to finding a stress function which satisfies Laplace's equation with given boundary values. The required function is then built up by a continuous distribution of singularities along the boundary. Consider the function $F \cos \theta / r$, where r, θ are polar co-ordinates with the pole at a point in the boundary, and F is a constant. This function satisfies Laplace's equation. By superposing a continuous distribution of such functions, for each point in the boundary with the proper value of the intensity F at each point, the boundary conditions can be completely satisfied. Due to the continuous distribution there need not be any singularities at the boundary for the complete solution.

In the practical application of this method, the value of F is determined at n discreet points along the boundary, by the successive approximation solution of n simultaneous algebraic equations, and is taken to vary linearly between these chosen points.

Complete solutions, including a plot of the contours of the stress function, are given for bending of a square-cornered T section and of a rolled I beam, as well as for torsion of a rolled-angle section, of a circular shaft with a square central hole or an eccentric circular hole, and for torsion in a square-cornered tube. Solutions are also given for shear stress concentrations produced by V, rectangular, and elliptic notches. The presence of infinite stress concentrations at the boundary does not prevent application of the method, because of the singular function which is employed.

No reference is made to any previous literature employing similar methods, nor are comparisons made with possible alternate procedures such as relaxation. D. C. Drucker, USA

411. Chih-Bing Ling, "On the stresses in a plate containing two circular holes," *J. appl. Phys.*, Jan. 1948, vol. 19, pp. 77–82.

A theoretical solution is presented to the problem of a plate containing two circular holes of equal size. The solution is accomplished by the method of adding to the given stress system a suitable biharmonic function which gives no stress at infinity. The parametric coefficients involved in the solution are adjusted so as to justify the boundary conditions at the edges of the holes. Bipolar co-ordinates are used. Three fundamental stress systems are discussed, namely the case of all-around tension, the case of longitudinal tension, and the case of transverse tension. Formulas for the stresses along the edges of the holes are derived and the values of the maximum stress are calculated.

The author does not seem to have known of the doctor-thesis of K. J. Schulz ["On the stress conditions in a plate through which holes have been bored (Over de spanningstoestand in doorboorde platen)," University of Delft, 1941]. In the latter paper another type of solution is given for this problem, which has the advantage of being extendible to three or more, or infinite rows of loaded or unloaded holes. R. G. Boiten, Holland

Experimental Stress Analysis

(See also Revs. 416, 454)

412. W. R. Campbell and A. F. Medbery, "Performance tests of wire strain gages—V. Error in indicated bending strains in thin sheet metal due to thickness and rigidity of gage," *Nat. adv. Comm. Aero. Tech. Note No. 1318*, July 1947, pp. 1–11.

The authors tested fifteen types of single-element multistrand wire strain gages, cemented to thin sheets loaded as cantilever beams. The indicated strains were always too large, the error increasing as the thickness of sheet decreases. For a thickness of 0.026 in., one of the gages showed an error of 40 per cent. This is attributed by the authors to the finite thickness of the gage, but no explanation is given of the large difference in error between gages of different types.

The standard used for the calculation of errors was the Tuckerman optical gage which the authors found to give results consistent to 1 per cent. A. J. Durelli, USA

413. Lawrence Lee Rauch, "Electronic commutation of strain gages for telemetering," *Proc. Soc. exp. Stress Anal.*, vol. 5, no. 1, pp. 111–121.

In pilotless flight testing it is necessary to transmit test data to a ground station to obtain a permanent record. These data are invaluable in finding the cause of a crash.

If the number of signals to be transmitted is large, they are synthesized into a single signal, transmitted by a single radio link to the ground, and analyzed into the original signals after reception. The paper discusses and describes such a telemetering system in which the above multiplexing method is used.

Nicholas Sag, Australia

414. G. B. Greenough, "Residual lattice strains in plastically deformed metals," *Nature, Lond.*, Aug. 23, 1947, vol. 160, p. 258.

In this note, the residual lattice strain found from x-ray reflections of a plastically deformed metal is stated to be the mean of the strains of grains with particular orientations, and not the mean strain of all grains. Consequently, the value is in general not zero but depends upon the choice of the reflecting plane. The author submits experimental evidence obtained by x-ray diffraction methods as substantiation. George Gerard, USA

415. A. G. H. Dietz and W. H. Campbell, "Bonded-wire strain-gage techniques for polymethyl methacrylate plastics," *Proc. Soc. exp. Stress Anal.*, vol. 5, no. 1, pp. 59–62.

This paper reports the development of suitable cements for bonding electric-resistance strain gages to certain plastics whose characteristics are adversely affected by the application of the usual cements.

A few representative hysteresis measurements on polymethyl methacrylate are also included. Henry W. Foster, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 409, 410, 428, 443, 453, 545)

416. A. Weigand, "Determination of the stress-concentration factor of a stepped shaft stressed in torsion by means of precision strain gages," *Nat. adv. Comm. Aero. Tech. Memo. No. 1179*, Sept. 1947, pp. 1–12 (trans. from *Luftfahrtforsch.*, 1943, vol. 20, p. 217).

By means of an electric precision strain gage of 1.3-mm gage length, the author has investigated the factors of stress concentration at the fillets of stepped shafts subjected to torsion, covering the range $0.1 \leq r/d \leq 0.25$ and $1.11 \leq d/D \leq 2$. He concludes that in this range the results are in good agreement with the Sontag formula

$$K = \frac{1}{D}\left[3/2\,\frac{(d+2r)\,(d+4r)}{d+6r} + \frac{(D-d-2r)(d+12r)}{12r}\right]$$

The author's results are in good agreement with those of Jacobsen ["Design Data, Applied Mechanics, Book I," Amer. Soc. mech. Engrs., 1944, p. A-27] where they overlap and may be considered as an extension of Jacobsen's curves for the range $r/d = 0.12$ to 0.25. M. M. Leven, USA

417. R. L. Moore, "Observations on the behavior of some noncircular aluminum-alloy sections loaded to failure in torsion," *Nat. adv. Comm. Aero. Tech. Note No. 1097*, Feb. 1947, pp. 1–47.

These tests constitute an extension of a previous series on 24S-T noncircular bars and tubing [*Nat. adv. Comm. Aero. Tech. Note No. 855*, 1943]. Torsion-test data are reported for 24S-T and X74S-T, alloy-machined to the shape of a cruciform, and an I section, in which shearing stresses were of principal importance, together with extruded flanged members of I, Z, and channel section, for which significant longitudinal stresses

were also developed. The effects of restrained and simply supported ends were compared and the behavior was studied up to rupture.

Reasonable agreement was found between the measured and computed behavior in the elastic range. Twist about an axis other than the shear center resulted in added transverse shear and bending stresses. Ultimate torques for flanged sections ranged from 4.2 to 7.3 times the torque computed from formulas applicable in the elastic range to develop the shear strength.

Thomas J. Dolan, USA

418. A. Rinkert, "Continuous beams on elastic supports (Kontinuerliga balkar på elastiska stöd)," *Tekn. Tidskr.*, Nov. 22, 1947, vol. 77, pp. 875–884.

The author develops new diagrams and tables for the influence functions of the statistically indeterminate vertical reactions in the elastic supports of a beam of two, three, four and five equal spans. The tables apply primarily for the case when each inner support has the same vertical displacement per unit force. However, a method is suggested by which the tables can be used also in cases when this quantity is different for the various supports, provided that their arrangement is a symmetrical one.

Paul Neményi, USA

419. A. K. Rukdadze, "Influence of transverse shear on the bending of a bar" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), May-June 1947, vol. 11, pp. 351–356.

The author applies to this problem the generalized "nonlinear" theory of elasticity which was initiated by L. N. G. Filon and F. Murnaghan [*Amer. J. Math.*, 1937, vol. 59, no. 2]. He refers to work done and published in Russia from 1938 to 1944 on particular solutions of problems of nonlinear theory of elasticity, in which the elastic constants of the classic theory are kept unchanged and additional elastic constants are introduced to describe the deviations from linear behavior.

On the basis of this nonlinear theory the author gives the differential equations of equilibrium, the compatibility conditions and the boundary conditions for the present problem. In this way he derives rather complicated formulas for the bending stresses in a bar, which take into account the influence not only of the bending moment, but also of the transverse shear force, which is not considered in classic linear bending theory.

M. T. Huber, Poland

420. N. H. Arootyunyan, "Torsion of an elliptical ring sector" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), Sept.-Oct. 1947, vol. 11, pp. 543–546.

This paper concerns an anisotropic shaft, through each point of which three perpendicular planes of elastic symmetry pass. The author takes advantage of a special system of curvilinear co-ordinates, which was introduced in his work published in the same periodical in 1942. The comparison of his results with those of St. Venant and Dinnik shows good agreement, especially for inner angles between $\pi/4$ and 2π.

The author states that a radial split in the elliptical section of a twisted shaft reduces its rigidity 1.87 times and that the same split in the elliptical ring reduces it 70 times.

Witold Wierzbicki, Poland

421. U. Richard, "Stretching of a heavy cable suspended from two fixed anchorages (Dilatazione di una fune pesante sospesa a due estremi fissi)," *R. C. Accad. Lincei*, Sept. 1947, vol. 3, pp. 321–325.

The author considers a uniform, homogeneous cable, of given length, suspended at the two ends, and in equilibrium under the action of its own weight. He shows that the determination of the components of the reaction at one end reduces to the solution of the well-known equation $\frac{\sinh \xi}{\xi} = \alpha$ (where ξ involves one component of the reaction and α is given), and obtains this solution by means of a series expansion.

The author then assumes that the cable undergoes a small change in length, and he develops explicit formulas for calculating the small changes in the reactions which this causes.

Ratip Berker, Turkey

Plates, Disks, Shells, Membranes

(See also Revs. 411, 427, 429, 432, 433, 434, 437, 438, 439, 440, 441, 442, 449, 450, 454)

422. V. Z. Vlasov, "On two representations of equations of a spherical shell" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), Sept.-Oct. 1947, vol. 11, pp. 521–526.

The author's starting point is his equations published in the same periodical in 1946. Certain hypotheses having a geometrical character allow consideration of the problem of a thin elastic shell as a two-dimensional problem. Neglecting transverse shearing deformations and deformations in the direction of the normal to the inner surface of the shell and introducing bending and torsion parameters which depend only on the displacement w normal to the surface of the shell, the author obtains a system of nineteen differential equations which completely cover the general shell problem. For the spherical shell, this system of equations becomes considerably simplified.

The second method of representing the equations of the shell relies on the introduction of a stress function φ, which plays a role somewhat similar to that of the Airy function in the plane elasticity problem. The forces and the moments of the shell are first expressed in terms of the functions φ and w, and these in turn are then expressed in terms of a single function Φ.

Witold Wierzbicki, Poland

423. S. S. Manson, "The determination of elastic stresses in gas-turbine disks," *Nat. adv. Comm. Aero. Tech. Note No. 1279*, May 1947, pp. 1–30.

The gas-turbine disk is analyzed as a circular elastic plate with variable thickness in the radial direction, the stresses due to centrifugal force being computed. It is assumed that there may be variations in the radial direction of: plate thickness, temperature, coefficient of thermal expansion, Young's modulus, Poisson's ratio and material density. The differential equations of elasticity are replaced by finite-difference equations, which are solved, with the boundary conditions, to obtain the stresses.

Numerical examples are given of a solid disk, a disk with a central hole, and a disk having the outer portion attached to the inner portion by shrink fitting and welding.

Stanley U. Benscoter, USA

424. A. L. Goldenweiser, "Membrane theory of shells whose middle surface is a curve of the second order" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), Mar.-Apr. 1947, vol. 11, pp. 285–290.

This paper deals with shells, the middle surface of which is an ellipsoid, a hyperboloid of two sheets, an elliptic paraboloid, a hyperboloid of one sheet, or a hyperbolic paraboloid. The de-

termination of the membrane stresses T_1, T_2, S_1, and S_2, is reduced to the integration of a system of equations, which for shells of a positive Gauss curvature leads to integration of Poisson's equation, and for shells of negative Gauss curvature to the heterogeneous wave equation.

The author demonstrates that the equations derived can also be applied to other shells satisfying certain geometrical conditions identical to the given ones in the vectorial formula for shells of second order. By using new variables as functions of the displacements the author deduces a system of differential equations in terms of these displacements. M. T. Huber, Poland

425. I. N. Vekua, "Some fundamental questions in the theory of a thin spherical shell" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*), Sept.-Oct. 1947, vol. 11, pp. 499–516.

This paper concerns the integration of the equilibrium equations of a thin spherical shell. The author expresses the forces, moments and displacements in the shell in terms of four arbitrary holomorphic functions. These equations, containing the components of the internal and external forces and moments, are first referred to an isothermal set of curvilinear co-ordinates, using the work of Love. They are then combined with the relations between the forces or moments and the displacements, to obtain equations involving only the components of the displacements, and finally involving only the displacement normal to the surface.

The author applies the theory to the case of a shell of the shape of a spherical segment fixed along its perimeter. When the shell is shallow, the calculation can be greatly simplified.

Witold Wierzbicki, Poland

426. B. R. Seth, "Bending of clamped rectilinear plates," *Phil. Mag. London*, Apr. 1947, vol. 38, pp. 292–297.

This paper deals with the deflection of a regular n-gonal plate under uniform pressure, the edges of the plate being clamped. The interior of the n-gonal region is mapped upon the interior of a unit circle by means of the Schwarz-Christoffel transformation. The integrand of the integral representing this transformation is then expanded into an infinite series and is integrated term by term, thus yielding the transformation as an infinite series. The real solution to the inhomogeneous biharmonic equation is expressed in terms of complex and conjugate complex functions of points in the n-gonal region. By using the transformation from the n-gonal region to the unit circle, this solution is expressed as a double infinite sum in which the variables are the polar co-ordinates in the unit circle.

The introduction of the boundary conditions yields an infinite set of simultaneous linear equations in the undetermined coefficients of the double infinite sum which expresses the deflection of the n-gon. By approximating the solution to this infinite set of equations the author is able to compute the approximate central deflection and bending moment at the center of the clamped edge of the regular n-gon. His results agree closely with those of Hencky and others. A particularly interesting case is shown in his upper bound solution in which the number of sides of the n-gon becomes infinite and the clamped plate becomes circular. Henry J. Barten, USA

Buckling Problems

427. Livio Norzi, "On the general theory of elastic instability (Sulla teoria generale dell' instabilità elastica)," *G. Gen. civ.*, Sept.-Oct. 1947, vol. 85, pp. 414–420.

The author considers the influence of a small displacement u,v,w, for a body in a state of initial stress $\sigma_x, \ldots \tau_{yz}$. The potential energy E is split up into three parts $E = L + S - F$, where L is the elastic energy, corresponding to the displacement, S is the work done by the stress sytem against the displacement, and F the work done by external forces. If $(w_x, w_y, w_z) = curl\ (u, v, w)$, then the expression for S comprises a volume integral, the integrand of which is of the form $U = [\sigma_x(w_y^2 + w_z^2) + \ldots -2\ \tau_{yz}\ w_yw_z]/2$.

The external forces being multiplied with a "factor of stability" φ, instability with respect to u, v, w occurs in the case $E = L + \varphi(S-F) = 0$ for a value of $\varphi > 0$. For constant external forces we have $F = 0$ and in this case a sufficient condition for instability will be $U > 0$. In applying this and using the methods of the calculus of variations, the author checks the results for struts and plates found by Bryan and Timoshenko, and obtains some new results for a cylindrical shell compressed in the axial direction.

Folke K. G. Odqvist, Sweden

428. F. Vasilesco, "On buckling of straight beams having a constant cross section and a variable moment of inertia (Sur le flambement des poutres droites à section constante et à moment d'inertia variable)," *C. R. Acad. Sci., Paris*, Oct. 27, 1947, vol. 225, pp. 716–718.

This paper gives a general mathematical solution of the buckling problem under certain special conditions.

The result is in the form of an equation which for practical problems appears to be more difficult to solve than to obtain an answer by one of the standard approximate methods in common use. N. M. Newmark, USA

429. M. Holt and G. W. Feil, "Comparative tests on extruded 14S-T and extruded 24S-T hat-shape stiffener sections," *Nat. adv. Comm. Aero. Tech. Note No. 1172*, Mar. 1947, pp. 1–17.

Test data are presented which furnish comparisons of the column strengths of flat plate-stringer panels with 14S-T and 24S-T stringers. The specimens with 14S-T stringers were somewhat stronger than those with 24S-T stringers, due to the higher compression yield stress of 14S-T material.

H. L. Langhaar, USA

430. P. Cicala, "On the analysis of small deformations in the elasto-plastic field (Sull'analisi delle piccole deformazioni nel campo elastoplastico)," *R. C. Accad. Lincei*, Sept.-Oct. 1947, vol. 3, ser. 8, pp. 325–329.

In problems of elastic stability for structures manufactured out of such materials as light alloys or stainless steels, there is often considerable deviation from Hooke's law, which the author believes should be taken into account by using the classical theories of plasticity. He derives formulas for small variations in the state of deformation according to the theories of von Mises, Reuss, and Hencky, and suggests two new experiments with thin-walled cylindrical tubes, which would be capable of distinguishing between these theories.

Folke K. G. Odqvist, Sweden

431. G. L. Gallaher and R. B. Boughan, "A method of calculating the compressive strength of Z-stiffened panels that develop local instability," *Nat. adv. Comm. Aero. Tech. Note No. 1482*, Nov. 1947, pp. 1–15.

A method is presented for the determination of the critical

buckling stresses and the average compressive stresses at maximum load for thin sheet panels stiffened with rigidly attached Z-section stringers. The results are given in terms of the usual equations for the buckling of plates, in which the edge fixity coefficients are expressed as functions of the geometrical parameters of the panel. The values of the coefficients are given in tables and curves for various panel proportions. The secant modulus is used when the critical stresses are higher than the proportional limit.

Curves are also presented which show that the local instability stresses plotted against strains follow closely the compressive stress-strain curves of the material, for 24S-T Z-stiffened panels.

M. V. Barton, USA

432. N. F. Dow and W. A. Hickman, "Design charts for flat compression panels having longitudinal extruded Y-section stiffeners and comparison with panels having formed Z-section stiffeners," *Nat. adv. Comm. Aero. Tech. Note No. 1389*, Aug. 1947, pp. 1–76.

Some column tests were performed upon 24S-T and 75S-T plate-stringer panels that were constructed by riveting flanged, extruded Y-section stringers to flat sheets, with the stems of the Y's extending outward. The test data were used to construct design charts to enable airplane designers to determine quickly dimensions for minimum weight of a plate-stringer panel of given length and end fixity which will carry a given average compression load per unit width. Comparisons with similar charts for Z-type plate-stringer panels show that the Y-section stringers are more efficient when the sheet is thin, and that the Z-section stringers are slightly more efficient when the sheet is thick.

H. L. Langhaar, USA

433. N. F. Dow and W. A. Hickman, "Effect of variation in diameter and pitch of rivets on compressive strength of panels with Z-section stiffeners. Panels of various lengths with close stiffener spacing," *Nat. adv. Comm. Aero. Tech. Note No. 1421*, Sept. 1947, pp. 1–18.

Some 24S-T plate-stringer panels with Z-section stringers were tested to determine the effect of rivet size and spacing upon the column strengths of such panels. For the short panels, in which local crippling occurred, the strength was improved by increasing the number or the size of the rivets, but for the long panels, in which column bending occurred, an increase in number or size of the rivets had a slightly adverse effect.

H. L. Langhaar, USA

434. W. A. Hickman and N. F. Dow, "Compressive strength of 24S-T aluminum-alloy flat panels with longitudinal formed hat-section stiffeners having a ratio of stiffener thickness to skin thickness equal to 1.00," *Nat. adv. Comm. Aero. Tech. Note No. 1439*, Sept. 1947, pp. 1–20.

Column test data are given for a number of 24S-T plate-stringer panels with formed hat-section stringers of the same thickness as the plate. The work is a part of a more extensive program of tests of hat-type plate-stringer combinations, which will furnish data for design charts for such panels.

H. L. Langhaar, USA

435. P. C. Hu and J. C. McCulloch, "The local buckling strength of lipped Z columns with small lip width," *Nat. adv. Comm. Aero. Tech. Note No. 1335*, June 1947, pp. 1–15.

A method, which makes extensive use of previous publications, is given for calculating the local buckling strength of lipped Z columns with small lip width, since the usual method does not apply to this type of Z column. One numerical example is presented in detail, and the results are compared with experiment and show satisfactory agreement.

A. E. Green, England

436. Manuel Stein and John Neff, "Buckling stresses of simply supported rectangular flat plates in shear," *Nat. adv. Comm. Aero. Tech. Note No. 1222*, Mar. 1947, pp. 1–13.

The shear buckling stresses are evaluated for flat rectangular plates with simply supported edges. Symmetrical buckling patterns (odd number of half waves) as well as antisymmetrical (even number of half waves) are considered. Comparison is made between the critical stresses for the symmetrical and antisymmetrical modes, and it is shown that for some length-to-width ratios the critical stresses are about 4 per cent lower for the antisymmetrical pattern than for the symmetrical pattern assumed by previous investigators.

A curve is given for the determination of the critical stresses from the plate dimensions.

M. V. Barton, USA

437. S. B. Batdorf and Manuel Stein, "Critical combinations of shear and direct stress for simply supported rectangular flat plates," *Nat. adv. Comm. Aero. Tech. Note No. 1223*, Mar. 1947, pp. 1–29.

The buckling of simply supported rectangular flat plates under combinations of shear and direct stress is investigated by means of the Rayleigh-Ritz method of variation of energy. The deflection function is expressed in the form of a two-dimensional Fourier series, the nonvanishing coefficients of which define the combinations of shear and direct stress for which the system is in unstable equilibrium. The application of the energy-variation method results in an infinite set of linear equations in the infinite number of Fourier coefficients, of which the first ten were used in the expansion of the deflection function. In solving the equations, the matrix-iteration method developed by von Kármán-Biot was used, because of the nonconvergence of the conventional iteration method.

The paper is accompanied by six diagrams relating the ratio of shear stress to critical stress in pure shear, and the ratio of direct stress to critical stress in compression, for various length-width ratios of the plate.

A. M. Freudenthal, USA

438. S. B. Batdorf, M. Schildcrout, and M. Stein, "Critical stress of thin-walled cylinders in axial compression," *Nat. adv. Comm. Aero. Tech. Note No. 1343*, June 1947, pp. 1–21.

This paper uses three parameters, involving $\sigma t L^2/D$, $L^2/(rt)$ and the radius-thickness ratio r/t, to interpret test data on thin-wall cylinders loaded in axial compression. As the length L approaches zero the value of k approaches 4 for clamped edges, and 1 for simply supported edges, as indicated by theory. For values of length-radius ratio $L/r > 0.75$ the buckling stress reduces to $\sigma = CEt/r$, where C varies from 0.22 for $r/t = 500$ to 0.078 for $r/t = 3000$.

R. M. Wingren, USA

439. S. B. Batdorf, M. Stein, and M. Schildcrout, "Critical combinations of torsion and direct axial stress for thin-walled cylinders," *Nat. adv. Comm. Aero. Tech. Note No. 1345*, June 1947, pp. 1–36.

The partial differential equation defining the combinations of shear and axial stress which will cause a cylinder to buckle is

solved by the Galerkin method. The solution is carried out for cylinders with simply supported and clamped edges.

The results of the theory are modified in order to bring the test data into agreement. Empirical curves and formulas based upon this modified theory are derived for design purposes. The results may be used over a wider range of cylinder dimensions than was possible with previously available results.

Raymond L. Bisplinghoff, USA

440. S. B. Batdorf, M. Schildcrout, and M. Stein, "Critical shear stress of long plates with transverse curvature," *Nat. adv. Comm. Aero. Tech. Note No. 1346*, June 1947, pp. 1–23.

This is one of a recent series of NACA notes, on the buckling of thin shells under various loading conditions, which are summarized and discussed in one of these notes [S. B. Batdorf, *Nat. adv. Comm. Aero. Tech. Note No. 1342*, 1947]. The present note gives the details of the solution for the buckling load of long plates with transverse curvature under shear, with simply supported or clamped edges. The results are given conveniently by a simple formula and by curves. The validity of the approach is established in the above reference. Bruno A. Boley, USA

441. S. B. Batdorf, M. Schildcrout, and M. Stein, "Critical combinations of shear and longitudinal direct stress for long plates with transverse curvature," *Nat. adv. Comm. Aero. Tech. Note No. 1347*, pp. 1–36.

A theoretical solution is presented for the buckling stresses of long plates with transverse curvature subjected to shear and longitudinal direct stress. The theoretical critical-stress combinations for plates, having either simply supported or clamped edges, are given in figures and tables, and a comparison is made with a previous theoretical solution for simply supported plates.

In the compression range, the theoretical curves are unsuitable for use in design because long plates with substantial curvature, loaded in axial compression alone, buckle at stresses that are much less than the theoretical values of critical stress. An investigation was therefore made to determine the modifications required to make the theoretical curves compatible with the available experimental data on plates in axial compression. Interaction curves based upon this investigation are provisionally recommended for use in design. R. G. Boiten, Holland

442. S. B. Batdorf, M. Stein, and M. Schildcrout, "Critical shear stress of curved rectangular panels," *Nat. adv. Comm. Aero. Tech. Note No. 1348*, May 1947, pp. 1–29.

A solution based upon the classical small-deflection theory is given for the critical shear stress of curved rectangular panels with simply supported edges. Two sets of computed curves are given, one for panels having curved sides longer than the straight sides, and the other for panels having straight sides longer than the curved sides.

By comparing the computed results of the simply supported panels with available results for cylinders and long curved strips and flat panels, it is suggested that the critical shearing stresses for curved rectangular panels with clamped edges may be estimated, and two sets of such curves are included.

A comparison between the theoretical values and the available experimental ones indicates that the experimental data are in general considerably below the theoretical curves. This phenomenon can perhaps be explained by nonlinear large-deflection theory as was attempted by von Kármán and Tsien [*J. aero. Sci.*, 1939, vol. 7, pp. 43–50 and pp. 276–289].

C. T. Wang, USA

Joints and Joining Methods

(See also Revs. 409, 466)

443. F. Vogt, "The distribution of loads on rivets connecting a plate to a beam under transverse loads," *Nat. adv. Comm. Aero. Tech. Memo. No. 1134*, Apr. 1947, pp. 1–24 (reprint of *Roy. Aircr. Est. Rep. No. SME 3301*, 1944).

The author analyzes the load distribution in a beam reinforced by a plate riveted to its bottom along its entire length. Due to slip of the rivets the plate is stressed less than simple bending theory would indicate. This effect is especially great when large transverse shear is present or yielding of the rivets occurs.

The numerical examples are calculated using arbitrarily assumed rivet stiffnesses, so that they are not directly applicable to practical problems. The case investigated also differs from the practically important case when the reinforcement does not extend over the entire beam length, for which the reviewer has found that the reinforcement is not only relatively ineffective but may overload the end rivets. F. Stussi, Switzerland

444. F. Vogt, "The load distribution in bolted or riveted joints in light-alloy structures," *Nat. adv. Comm. Aero. Tech. Memo. No. 1135*, Apr. 1947, pp. 1–39 (reprint of *Roy. Aircr. Est. Rep. No. SME 3300*, 1944).

This is a theoretical study of load distribution in series-riveted and bolted joints. For double-shear joints the load distribution can be calculated from the condition that the difference between the displacements of two successive rivets must match the difference between the elastic strains in the plate and in the straps, over the distance between the rivets. Previous studies, restricted to loads in the elastic range, are extended to loads above this limit.

The only remaining unknown is the displacement per unit load of the bolts, that is, their "stiffness." For light alloys the only available data are from tests of O. Volkerson [*Luftfahrtforsch*, 1938, vol. 15, p. 41] which, however, are not in good agreement with the author's theory.

For single-shear joints the load distribution depends also on the bending of the plates and thus on the magnitude of load. Systematic experimental and theoretical investigation is still needed to clarify this problem. F. Stussi, Switzerland

445. C. C. Fishburn, "Strength and slip under load of bent-bar anchorages and straight embedments in Haydite concrete," *J. Amer. concr. Inst.*, Dec. 1947, vol. 19, pp. 289–305.

This is a systematic study of the influence of bar shapes (plain or deformed), of their embedded length and of their bend angle on the bond between steel and concrete. A total of 44 different bends was tested, and the load and slip of the bars were recorded and represented in tables and diagrams. The conclusions drawn from the tests confirm previous tests in the field and the general practice in construction. The quantitative results will prove useful to research people as well as designers.

A. J. Durelli, USA

446. R. L. Templin and Marshall Holt, "Static and fatigue strengths of welded joints in aluminum-manganese-alloy sheet and plates," *Welding Res. Supplement*, Dec. 1947, vol. 12, pp. 705–711.

The following tests of butt- and spot-welded joints in ALCOA 3S sheet and plate are reported: (*a*) Strength of butt joints at temperatures varying from 600 F to —320 F; (*b*) static shear strength of spot welds in sheets from 0.02 in. to 0.12 in. thick;

(c) reversed flexure fatigue strengths of butt joints, using from 10^4 to 10^8 reversals; (d) axial-load fatigue strength of butt joints with and without removal of the weld bead; (e) fatigue shear strength of spot welds in annealed and half-hard sheets 0.032 in. and 0.064 in. thick, using from one to more than 10^8 cycles of reversed stress.

The paper does not discuss the effect of the size of weld, which this reviewer considers the most important variable affecting the fatigue strength. The range of stress, expressed as a percentage of the average static shear strength, decreases with increasing weld size for a given fatigue life. However, on an actual load basis fatigue strengths are greater for large than for small welds. S. K. Ghaswala, India

447. W. M. Wilson and Chao-Chien Hao, "Residual stresses in welded structures," *Welding Res. Supplement*, May 1947, vol. 12, pp. 295–320.

This paper describes tests on welded plates made to determine, by relaxation methods, the magnitude of the residual stresses left after welding, and the effect of these residual stresses upon the tensile properties of the plates.

The specimens were either unwelded or butt-welded longitudinally along the middle. Thermal stresses parallel to the weld were approximately equal to the yield point of the steel, which was greater around the weld than elsewhere in the specimen. Due to this yield-point increase, the static strength of the welded plates was somewhat greater than that of similar unwelded plates, while the ductility was somewhat less. Elongation values for the welded plates increased as the plate width increased.

A small number of fatigue tests were made with and without longitudinal butt welds, all with the surfaces planed and draw-filed to remove surface irregularities at the weld and elsewhere. The fatigue strength was somewhat less for specimens with welds than for those without, and somewhat greater for welded specimens that had been stress relieved by prestressing to the yield point than for those not stress relieved. The difference did not exceed the scatter of the various tests.

Specimens with welded circular seams had biaxial residual stresses approximating the yield point within and adjacent to the circular seam. When statically tested, they failed outside of the vicinity of the weld at ultimate loads slightly greater than for similar unwelded specimens.

Mortimer F. Sayre, USA

448. E. H. Schuette and D. E. Niles, "Data on optimum length, shear strength, and tensile strength of age-hardened 17S-T machine-countersunk rivets in 75S-T sheet," *Nat. adv. Comm. Aero. Tech. Note No. 1205*, Mar. 1947, pp. 1–41.

Rivets $^1/_8$, $^5/_{32}$, and $^3/_{16}$ in. in diameter were driven in countersunk holes and then machined flush on the countersunk side to obtain a smooth exterior surface. These were then tested in shear, using customary methods, and in tension parallel to the axis of the rivet, using a special fixture which reduced the bending component of the stress to a minimum. Satisfactory results were obtained if the "buck" (the length of rivet protruding beyond the surface of the countersunk sheet) was kept between 0.9 and 1.5 times the rivet diameter.

Other conclusions, drawn by the reviewer from the test data given, are that the full shear strength of 40,000 to 45,000 psi of rivet cross section was developed if the plate thickness exceeded one third of the rivet diameter, and that under the test conditions used, the tensile strength reached a value of 75,000 psi of rivet cross section if the plate thickness exceeded two thirds of the rivet diameter. Mortimer F. Sayre, USA

Structures

(See also Revs. 396, 418, 444, 445, 448)

449. John E. Duberg and Joseph Kempner, "Stress analysis by recurrence formula of reinforced circular cylinders under lateral loads," *Nat. adv. Comm. Aero. Tech. Note No. 1219*, Mar. 1947, pp. 1–44.

This paper is a development of the work of Ebner and Koller on stresses in thin cylinders with ring and stringer reinforcement. Attention is confined to circular sections, and the stringers are assumed to be so close that they can be treated as continuous. Self-balancing loads, which correct the usual approximate distributions, are expressed as Fourier series around the periphery. The determination of the coefficients of these series is reduced to the solution of five-term recurrence relations. Explicit formulas are given for the coefficients of these relations, and for the "constant terms," when these arise from concentrated loads on the rings.

Solutions of the relations are presented for the case of uniform reinforcement, but they can be used to obtain approximate solutions for cases where the loaded ring is not too near to a fixed end, a cutaway, etc. Comparison is made with earlier experimental work, and good agreement is recorded. The formulas in the paper can form a basis for design calculations on heavily loaded rings in aircraft fuselages.

W. S. Hemp, England

450. Joseph Kempner and J. E. Duberg, "Charts for stress analysis of reinforced circular cylinders under lateral loads," *Nat. adv. Comm. Aero. Tech. Note No. 1310*, May 1947, pp. 1–60.

This paper presents coefficients for the stress analysis of reinforced circular cylinders, loaded in the plane of the reinforcing rings, to facilitate the calculation of shear and bending stresses in the cylindrical sheet as well as shears, thrusts, and bending moments in the rings. Charts are presented for basic concentrated radial, tangential, and bending loading conditions, stresses due to combinations of loads on one or more rings being obtained by superposition. The method used in developing the charts considers deformations of both the sheet and the rings.

John E. Goldberg, USA

451. Chi-Teh Wang and S. Ramamritham, "Stress analysis of noncircular rings for monocoque fuselages," *J. aero. Sci.*, Dec. 1947, vol. 14, pp. 707–712.

This paper presents a minor modification of the Column Analogy Procedure for analyzing closed rings under the action of plane forces. In applying the proposed method, the analyst estimates the location of three points of inflection in the closed ring and causes the static moment curve to pass through these points, thus attaining a condition of statical determinacy. The redundants are then calculated implicitly using the column analogy. Since the static curve is approximately correct for the closed ring, it follows that further corrections due to the redundants are small, and according to the authors increased accuracy is obtained. John E. Goldberg, USA

452. Pei-ping Chen, "Matrix analysis of pin-connected structures," *Proc. Amer. Soc. civ. Engrs.*, Dec. 1947, vol. 73, pp. 1475–1482.

The paper introduces and illustrates the application of matrix methods to the solution of pin-jointed frameworks which have

redundancy in the form of an excess of members or of reaction points. The procedure is to express the component loads at the joints as the product of a displacement matrix and a stiffness matrix for all members meeting in the joint. Equilibrium is established by summing for all joints, which gives a matrix equation in which a force matrix is equated to the product of a displacement matrix and a symmetrical stiffness matrix for the whole frame.

Usually, at hinged or roller reaction points, a total of three or more displacement components is zero so that all the elements in the corresponding rows and columns of the stiffness matrix drop out, leaving a much simplified matrix equation, giving the minimum number of simultaneous equations to be solved. It is this feature of the method which makes it particularly suitable for highly redundant pin-jointed space frameworks with multiple reaction points.

Since the analysis of the complete frame involves the solution of only one set of simultaneous linear equations of order equal to the number of unknown displacement components, the method is much neater than the piecemeal processes which the older methods necessitate, besides promising savings in labor in certain cases. Three simple examples are worked out to illustrate the procedure.

H. A. Wills, Australia

453. Edwin T. Kruszewski, "Bending stresses due to torsion in a tapered box beam," *Nat. adv. Comm. Aero. Tech. Note No. 1297*, May 1947, pp. 1–48.

When a hollow boxlike structure is subjected to torsion, the cross sections tend to warp out of their original planes. If a cross section is restrained against warping (as at the root of an airplane wing) axial strains are introduced. These must be added algebraically to those produced by bending, in the analysis of beams subjected to combined bending and torsion. The axial stresses caused by such torsional loading are sometimes referred to as "bending" stresses.

In a previous paper [Nat. adv. Comm. Aero. Advance Restricted Report, 1942] Kuhn presented a method for calculating the bending stresses due to torsion for a nontapered box beam. The present paper extends this analysis to include the tapered beam of rectangular cross section, in which the longitudinal elements, if extended, would meet at a point. Results of tests on one such beam were in good agreement with the theory. The effects of moderate taper are shown to be small.

F. R. Shanley, USA

454. Daniel Farb, "Experimental investigation of the stress distribution around reinforced circular cutouts in skin-stringer panels under axial loads," *Nat. adv. Comm. Aero. Tech. Note No. 1241*, Mar. 1947, pp. 1–21.

This paper describes experimental stress analysis of 24S-T aluminum panels, reinforced with longitudinal stringers and subjected to axial-tensile loads. Tuckerman strain gages and wire-resistance gages were used. The panels have circular holes of different sizes and the boundaries of these holes are reinforced with rings riveted to the panel. The distribution of stresses in the stringers and in the sheet is given in graphical form.

It was found that the maximum stress is not at the point of intersection of the transverse axis and the boundary of the hole, but on a stringer away from the hole on a line making approximately 45 deg with the transverse axis. Some empirical formulas are suggested for describing the results.

A. J. Durelli, USA

Plastic Flow, Failure

(See also Revs. 414, 417, 430, 464, 465, 466)

455. W. Prager, "On the interpretation of combined torsion and tension tests of thin-wall tubes," *Nat. adv. Comm. Aero. Tech. Note No. 1501*, Jan. 1948, pp. 1–11.

This paper deals with a proposed method of checking experimentally, by means of combined tension and torsion tests of thin-walled tubes, the validity of two basic theories of plasticity for materials with strain-hardening characteristics. Following recently established terminology, the author distinguishes between theories of "plastic deformation" and "plastic flow." This distinction rests on the character of the underlying stress-strain relations. According to the deformation theories, there is a one-to-one correspondence between the instantaneous states of stress and strain; according to the flow theories, at any given instantaneous state of stress there exists a nonintegrable one-to-one relation between the instantaneous rates of stress and strain.

The author formulates the general stress-strain relations corresponding to either theory for the particular states of stress and strain which arise in the tube tests under consideration. He then demonstrates that tests during which the ratio of axial loading vs. torque is maintained constant are inconclusive as regards a decision between the two types of theories. For the purpose of reaching such a decision, tests are advocated in which pure torsion is followed by axial tension at constant torque.

E. Sternberg, USA

456. H. Brandenberger, "Numerical calculation of strain hardening by means of cold stretching and compressing (Numerische Berechnung der Spannungsverfestigung beim Kaltrecken und Kaltstauchen)," *Schweiz. Arch.*, Aug. 1947, vol. 13, pp. 232–238; and Sept. 1947, vol. 13, pp. 268–275.

The author claims that the isotropic strain hardening and the Bauschinger effect, due to prior stressing in the plastic range, can be calculated from a residual stress distribution, which arises from a diphase nature of metal, and is macroscopically in equilibrium.

In order to obtain and describe the residual stress produced by plastic flow, the author uses, without so stating, the Lamé form of the stress-strain equations of isotropic *elasticity*. Calling the shear-modulus term by one name and the volume change term by another, he seems to confuse an arbitrary terminology with physical reality.

He states that upon unloading after tensile plastic flow, two equal and opposite isotropic residual stress systems are developed, positive volume stress terms balancing negative shear-modulus terms. In order to produce further plastic flow by tension in *any* direction, he further assumes that the applied strain must overcome the initial negative value plus the true elastic limit (strain hardening). On the other hand, flow under compression requires a shear-modulus term equal to the original elastic limit minus the initial magnitude (Bauschinger effect). Numerical calculations are made which compare very well with experiments on brass.

D. C. Drucker, USA

457. A. I. Gubanov, "Elementary deformations of elasto-viscous bodies" (in Russian), *J. tech. Phys.* (*Zh. tekh. Fiz.*), **1947, vol. 17, no. 4, pp. 475–490.**

The hydrodynamics of viscous fluids on the one hand and the theory of elasticity on the other, are regarded as the extremes of the general case of the elasto-viscous material. For the latter, both the shear modulus and the coefficient of viscosity must be

considered. The principles of the mechanics of such bodies were developed by Y. I. Frenkel ["Kinetic Theory of Fluids," publ. by Acad. of Sci., USSR, 1945, chap. 4]. The corresponding equations can be obtained by substituting for the shear modulus G in elastic theory the operator

$$\frac{\eta \frac{d}{dt}}{1 + \tau \frac{d}{dt}}$$

where η is the coefficient of viscosity, and $\tau = \eta/G$ is the reaction time.

On this basis the cases of elastic shear, torsion, and tension are generalized in this paper for elasto-viscous material. The elasto-viscous properties appear only in periodic or nonstationary regimes. Under stationary conditions simple viscous behavior obtains.

The following cases are treated in detail: (1) Shear in an infinite layer of finite thickness under (*a*) imposed displacements, (*b*) imposed forces, (*c*) tangential impact; (2) torsion of a cylindrical rod for (*a*) and (*b*) above; (3) uniform tension for (*a*) and (*b*) above.

The general equations take account of inertia effects. It is shown, however, that in most cases the magnitude of these effects is negligible, making simplifications possible. Numerical illustrations apply the theory to the behavior of a soil layer under shear, and to that of heated glass in various forming processes. It is stated that practical applications of this method are of interest in the field of polymers, plastics and glasses, and in geology, astrophysics, and the investigation of motion in highly viscous media. George Winter, USA

458. Livio Norzi, B. N. Clack, and N. L. Harris, "Spiral cracks in glass tubing," *Nature*, Mar. 1, 1947, vol. 159, pp. 306–307; Apr. 19, 1947, vol. 159, p. 541.

The experiments reported by Dr. J. J. Hopfield [*Nature*, 1946, vol. 158, p. 582] form the basis of these two letters. Norzi attempts a qualitative study of spiral cracks from static and dynamic considerations in the light of Griffith's classic theory of rupture of brittle materials.

Spiral cracks are also described by Clack and Harris, who observed that in thin-walled tubes with two varieties of unmatched glasses, spiral cracks developed when the circumferential tensile stress in the flashing glass reached 200 kg per sq cm (2850 psi).

S. K. Ghaswala, India

459. A. A. Ilyushin, "On the theory of plasticity in case of simple loading of plastic bodies with strain hardening" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, Mar.-Apr. 1947, vol. 11, pp. 293–296.

This paper presents a critical examination of the present status of the theory of plasticity. The author defines as "simple loading" a loading such that, as the loads increase, the local load intensities change at the same relative rate at all points.

He then briefly develops a single tensor equation of plastic behavior for "simple loading" and states that all current theories in this field are merely particular cases of this general equation. Specifically, he enumerates the terms which, when omitted from the general equation, result in the following theories of plasticity: (1) St. Venant-Levy-von Mises; (2) Prandtl-Reis; (3) Hencky-Nadai theory of small elasto-plastic deformations; (4) the new theory of Handelman-Prager, which was given in the preceding number of the same journal. This reduction of the Handelman-Prager theory to a special case of the tensor equation for simple loading is stated to prove that the latter theory does not contain functions which would make it applicable to, and bring it into agreement with, plastic behavior under "complex" loading.

In conclusion, the author proceeds to show by means of his tensor equation that, for "simple loading," all the enumerated theories become identical with and reduce to the simplest of them, the theory of small elasto-plastic displacements. He maintains that for this reason discussions regarding the relative accuracy of these various theories are without significance as far as "simple loadings" are concerned. As regards the "new Prager theory," it is maintained that it, likewise, cannot apply to "complex loadings" since it does not contain any terms which would reflect the Bauschinger effect, the rotation of principal axes, and strain hardening, a fact which would result in contradiction between theory and test in the case of "complex loading."

George Winter, USA

460. Benjamin Epstein, "Statistical aspects of fracture problems," *J. appl. Phys.*, Feb. 1948, vol. 19, pp. 140–147.

The author reviews various statistical theories of the strength of materials which have appeared in recent years. He shows how all those theories which are based on the concept of the weakest link are actually special cases and applications of an important general problem in the theory of statistics, the problem of determining the smallest value in samples of size n drawn from a population with a prescribed probability density function. The known solution of that statistical problem is summarized here, and its application to the problem of measuring the strength of material is illustrated.

The density function is associated with the number of flaws of various sizes or degrees of severity, and the smallest value is associated with the most severe type of flaw. By comparing the results of this study with the experimental observations of others, the author is able to indicate the type of distribution function which is natural in the theory of strength. Other applications of that statistical theory are mentioned, including those of determining electrical strength, the ability of a material to stop light rays and some cases of determining the life span of equipment.

R. V. Churchill, USA

461. E. M. Shevandin, "Effect of notches upon cold brittleness of steel—II. Calculation of cold brittleness for parts having notches" (in Russian), *J. tech. Phys. (Zh. tekh. Fiz.)*, Oct. 1947, vol. 17, no. 10, pp. 1119–1136.

The author proposes to determine the empirical dependance of the critical temperature T_k, for the brittleness of mild steel, on the form of the notch and on the velocity of the load. He assumes a characteristic parameter B, describing the effect of a notch of radius ρ at its bottom, in the form

$$B = \log\left(1/\sqrt{1 + \sqrt{a/\rho}}\right)$$

where a is the height of the "working" section beneath the notch.

Denoting the argument of the logarithm in the above formula as the "stress-concentration factor," he gives its dependence on the critical temperature in the form $1/\sqrt{1 + \sqrt{a/\rho}} = Ae^{C/T_k}$, where A and C are constants of the material. Finally the relation between the "loading velocity factor" v and the stress-concentration factor B at constant temperature is given in the form $v = Ee^{DB}$, where E and D are constants.

These relations are shown on a special three-dimensional diagram with the co-ordinate system $x = 1/T_k$, $y = \log v$, $z = B$,

which gives an arrangement of straight lines lying in a single plane. These straight lines pass quite accurately through a group of points determined by the author's experiments.

M. T. Huber, Poland

462. Alice Winzer and W. Prager, "On the use of power laws in stress analysis beyond the elastic range," *J. appl. Mech.*, Dec. 1947, vol. 14, pp. 281–284.

Recently A. A. Ilyushin pointed out that there is a proportionality between loads and stresses for a large class of nonlinear stress-strain relations, for all of which the secant shear modulus is proportional to a power of the octahedral shearing stress. He maintained that, by segments at least, any nonlinear stress-strain relation could be represented with sufficient accuracy by such power laws, and therefore that the stress components should increase at approximately the same rate as load, quite independently of the particular stress-strain law used in the analysis.

The authors were not convinced of the above conclusion, and proceeded to carry out a stress analysis for a rotationally symmetrical state of plane stress, produced in a large disk with a small circular hole by radial pressure uniformly distributed over the boundary of the hole. After working out the stress-strain relations, the authors show the manner in which the stress distributions differ for two different stress-strain laws. They therefore conclude that Ilyushin's power law must be used with great caution if reasonable accuracy is to be expected.

M. J. Manjoine, USA

463. V. V. Sokolovski, "Problems in statics of plastic and granulated materials" (in Russian), *Bull. Acad. Sci. USSR (Ser. techn. Sci.)*, Oct. 1947, pp. 1275–1286.

The author gives a review of methods for solutions of static plane problems of plastic and granulated materials, discussing his own work from his two monographs [issued by Acad. of Sci., USSR, 1942 and 1946].

In the first part of his paper the author investigates the case of plastic plane strain determined by the differential equations of equilibrium relating the stresses σ_x, σ_y, τ_{xy}, and by the condition of plasticity

$$(\sigma_x - \sigma_y)^2 + 4\tau^2_{xy} = 4k^2$$

where $k = \sigma_s/\sqrt{3}$ (Huber, von Mises), or $k = \sigma_s/2$ (St. Venant). The second part is devoted to the case of plastic plane stress. The third part considers the boundary equilibrium of granulated materials, making use of the work of F. Kötter and T. von Kármán. Finally, the author investigates the plastic torsion of a prismatic bar, giving a solution for an elliptical cross section, and for a bar consisting of two cylindric parts of different diameters connected end to end.

M. T. Huber, Poland

Design Factors, Meaning of Material Tests

(See also Revs. 461, 465, 468)

464. S. Timoshenko, "Stress concentration and fatigue failures," *Proc. Instn. mech. Engrs.*, 1947, vol. 157, no. 28, pp. 163–169.

This is the James Watt lecture delivered by the author on Apr. 25, 1947, at The Institution of Mechanical Engineers in London. The author reviews the theoretical and experimental determination of stresses and the fatigue failure of materials. He considers in particular the relation between stress concentration and fatigue failure, referring to tests made by Peterson and Wahl, and the importance of surface rolling as a means to improve fatigue strength of axles, referring to work by Föppl and by Horger.

The author also discusses the extension of photoelasticity to three-dimensional problems which he considers has not, as yet, yielded satisfactory results. The reviewer would note, however, that numerous applications have already been made using the freezing method, and that for many of them the error of the stress determinations is certainly not larger than the error of other experimental stress analysis methods.

A. J. Durelli, USA

Material Test Techniques

(See also Revs. 417, 470)

465. N. N. Davidenkov, "How to construct a stress-strain diagram by hardness measurements," *Metallurgia*, Dec. 1947, vol. 37, pp. 102–104.

The author proposes the construction of the complete tensile true stress-strain curve by an equivalent of Macgregor's two-load method. Two points on the assumed straight line are determined by translating indentation hardness into stress and strain, using Ludwik's data.

Three additional points are considered: (*a*) the yield stress, calculated from the diameter of the deformed zone around a cone-hardness impression; (*b*) the real breaking stress, which is determined from the scratch width in a scratch test, preferably using a wide scratch; and (*c*) the nominal ultimate strength. The modulus of elasticity is to be computed from a height-of-rebound test using an empirical conversion.

D. C. Drucker, USA

Mechanical Properties of Specific Materials

(See also Revs. 415, 417, 445, 446, 458)

466. R. D. Stout, L. J. McGready, C. P. Sun, L. F. Libsch, and G. E. Doan, "Effect of welding on ductility and notch sensitivity of some ship steels," *Welding Res. Supplement*, June 1947, vol. 12, pp. 335–357.

The effects of commercial variables on weld ductility of ship steels were measured by notch-bend tests at various temperatures. Transition temperature was used as a measure of weld ductility, although this temperature was often indefinite, particularly for hard metals. Variables considered were type of steel, heat input, power input, electrode type, aging time, postheating, multiple beads, hydrogen content, hardness, and grain size. Weld ductility was found to be improved by anything that lowered the maximum hardness in the weldment. The more ductile failures occurred because more deformation accompanied the propagation of visible cracks formed at a small strain.

J. D. Lubahn, USA

467. C. W. Briggs, "Recent developments concerning the properties of cast steels," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1948, vol. 70, pp. 37–47.

This is a summary of investigations of carbon and low-alloy steel castings, carried on at Carnegie Institute of Technology, Case School of Applied Science (now Case Institute of Technology), and Michigan College of Mines. Conclusions are based on results from several heats as cast by commercial foundries from production heats.

Comparative values of physical properties at room temperatures and down to —185 F for cast and wrought steels are given. The indications are that low-alloy steel castings will afford satis-

factory service in low-temperature applications. Impact values of quenched and tempered castings are appreciably higher than those of normalized castings with comparable ultimate strengths.

J. F. Snider, USA

468. L. Schapiro and R. H. Esling, "Stress notch sensitivity with eccentric holes," *Trans. Amer. Soc. mech. Engrs.*, Feb. 1948, vol. 70, pp. 135–138.

The stress notch sensitivity of aluminum alloys 24S-T, 75S-T, and 14S-T was determined experimentally for bars of rectangular cross section with holes of different diameters in different eccentric positions. By "stress notch sensitivity" the authors mean the effect of a notch, or any section discontinuity, upon the static ultimate stress of the material. The ultimate strength was computed as the maximum load divided by the initial net area, except for specimens that fractured first across the smaller section only; in the latter case, the first fracture load was used.

Plots are given of the relationship between notch strength, eccentricity of hole, and hole size. An equation is presented containing four constants that serve as indices of the stress notch sensitivity of the materials. The stress notch sensitivity of the three aluminum alloys is compared with the aid of these constants. The authors conclude that the relative stress notch sensitivity is the same for these materials with eccentric holes as had been reported for central holes or notches.

Frank Baron, USA

469. L. Feret and F. Caen, "Mechanical strength of masonries (La résistance mécanique des maçonneries)," *Bâtim. Trav. publ.*, Oct. 5, 1947, ser. I, no. 37, pp. 1–19.

This is a report of compression tests of sixteen brick pillars to determine the relation between the strength of bricks and mortars and the total strength of the pillar, as well as the distribution of stresses on the joints between layers. For the latter purpose the authors used a magnetic cell developed by Nils Hast in Sweden. The results are analyzed and compared with values given by previous researchers.

A. J. Durelli, USA

470. James A. Miller, "Stress-strain and elongation graphs for Alclad aluminum-alloy 75S-T sheet," *Nat. adv. Comm. Aero. Tech. Note No. 1385*, Nov. 1947, pp. 1–36.

This is a report on static tensile and compressive tests of the high-strength aluminum alloy 75S-T. Standard methods were used for the determination of the stress-strain diagrams. Photogrid measurements were made for the local elongation tests, using an engraving technique somewhat different from that of Brewer and Glassco. The graphs are presented in dimensionless form.

A. J. Durelli, USA

471. J. W. Freeman, E. E. Reynolds, and A. E. White, "An investigation of the high-temperature properties of chromium-base alloys at 1350 F," *Nat. adv. Comm. Aero. Tech. Note No. 1314*, May 1947, pp. 1–21.

Chromium-base alloys were tested which had rupture strengths as high as 54,500 psi for fracture in 1000 hours at 1350 F. This is 25 per cent higher than values previously published for other alloys under these conditions. Of the five different alloys tested the most promising seems to be 60 Cr–25 Fe–15 Mo with less than 0.05 per cent carbon and about 0.6 per cent silicon. The paper is illustrated with microphotographs.

A. J. Durelli, USA

Soil Mechanics; Seepage

(See also Revs. 457, 463)

472. B. F. Kazarnovskaya, "Movement of water-oil interface and water encroachment into wells under hydrostatic head" (in English), *C. R. Acad. Sci. URSS*, Mar. 20, 1947, vol. 55, no. 8 pp. 689–692.

The author investigates as a two-dimensional problem of potential flow, the lateral intrusion of water into an inclined stratum of oil, during pumping from an array of equally spaced wells. Differences in density and viscosity are neglected. The resulting complex relationship permits solution for the form of the water-oil interface and the relative proportions of water and oil withdrawn as a function of time, boundary geometry, and permeability.

Hunter Rouse, USA

473. V. V. Vedernikov, "A physical picture of free seepage" (in English), *C. R. Acad. Sci. URSS*, Jan. 30, 1947, vol. 55, no. 3, pp. 199–202.

A general physical explanation is given of the seepage of water in soil with its surface exposed to the atmosphere, based on the author's analysis of model experiments. Illustrations are included, using a canal with various subsurface capillary-water and ground-water conditions. It is emphasized that the movement of water in the capillary-aerial zone is always toward smaller heads regardless of whether this is toward higher or lower moisture content.

C. M. Duke, USA

Potential Flow of Incompressible Fluids

(See also Revs. 457, 472, 487, 509, 510, 512, 515, 524, 537, 538, 546)

474. L. Castoldi, "On a property of steady motion of incompressible fluids in which the streamlines form a normal congruency of lines of equal velocity (Sopra una proprietà dei moti permanenti di fluidi incomprimibili in cui le linee di corrente formano una congruenza normale di linee isotache)," *R. C. Accad. Lincei*, Sept.-Oct. 1947, vol. 3, ser. 8, pp. 333–337.

The potential-flow problem studied in this paper concerns the condition under which an incompressible three-dimensional flow, fulfilling the continuity conditions, has streamlines along which the velocity is constant.

The result is obtained that surfaces normal to the streamlines must be minimum surfaces, or in other words, that the curvature must vanish. Study is then made of the additional condition required for maintenance of motion by a conservative-force system.

Frank L. Wattendorf, USA

475. Gabriel Viguier, "Flow of a viscous fluid with high temperature and velocity gradients (L'écoulement d'un fluide visqueux avec gradients de température et gradients de vitesse élevés)," *C. R. Acad. Sci., Paris*, Apr. 9, 1947, vol. 224, pp. 1048–1050.

The author investigates the effect of temperature gradients on the flow of a viscous fluid, considering coefficients of viscosity as supplementary variable parameters. He treats the steady-state motion of an incompressible fluid in two particular cases: (*a*) flow of the fluid between two plates; and (*b*) flow between two coaxial cylinders, one fixed, the other rotating uniformly. For the latter case he obtains the velocity at any point as $C_2\rho - 0.707C_1/\rho$,

where C_1, C_2 are constants and ρ is the radius of the point. The assumption of constant viscosity would give $C_2\rho - 0.75C_1/\rho$, while classical mechanics gives $C_2\rho - 0.5C_1/\rho$.

Ahmed D. Kafadar, USA

476. Jean-Jacques Moreau, "Behavior at infinity of slow steady flow (Sur l'allure à l'infini d'un écoulement permanent lent)," *C. R. Acad. Sci., Paris*, May 28, 1947, vol. 224, pp. 1469–1472.

For Stokes' flows, with the fluid at rest at infinity, it is usually assumed that: (1) the velocity tends to zero at least as fast as the inverse first power of the radial distance, and (2) the pressure tends to a constant value, the excess over this value tending to zero at least as fast as the inverse square of the distance. The author gives a deduction of these results based on the single assumption that the velocity tends to zero at infinity.

C. C. Lin, USA

477. J. Giltay, "Laminar flow in a slightly curved round tube (Over de laminaire vloeistofstroming in een zwak gekromde ronde buis)," *Ingenieur's Grav.*, Oct. 17, 1947, vol. 59, pp. A337–338.

The equations are set up for laminar flow in a circular pipe, the axis of which is curved with a large radius. The resulting flow is due to the superposition of: (1) a primary flow, analogous to that which takes place in a straight pipe, having a paraboloidal velocity distribution, (2) a secondary or "perturbing" flow, following the equations of Stokes and Navier, in which the internal force is a centripetal force.

The velocity components of this flow are computed and some characteristics are derived. Alb. Schlag, Belgium

478. Alexander Weinstein, "On axially symmetric flows," *Quart. appl. Math.*, Jan. 1948, vol. 5, pp. 429–444.

The author presents an extension of the theory of potential flow of an incompressible fluid about a body of revolution. Previous flow cases have been confined to a uniform flow plus sources, sinks, line sources, and line sinks along the axis of symmetry. Axially symmetric distributions of sources and sinks along circles, disks, and cylinders have also been considered, with expressions for the Stokes' stream function in terms of Bessel's functions.

It is known from the theory of the development of potential functions in series that an axially symmetric function is determined uniquely by the values it takes along a segment of the axis of symmetry. In this paper the value of a potential function on the axis of symmetry due to a uniform distribution of sources on a circular ring is obtained by elementary means. The harmonic function which reduces to these values on the axis is then found. The conjugate function, the Stokes' stream function, is next obtained.

By integration of the resulting stream function over the surface of a disk, and throughout a cylinder, the stream functions for these cases are obtained. The stream function for a disk, in the same form, has been given by Lamb ["Hydrodynamics," Cambridge Univ. Press, 6th ed., art. 102]. The author examines the resulting expressions for stream function, which are different for the two sides of the ring, showing that the Stokes' stream function is multiple-valued.

By superposing a uniform flow parallel to the axis of symmetry upon the source flow distributed over the surface of a disk, the flow around a new shape of half body is obtained, subject to certain restrictions of source distribution and strength. The point of stagnation is determined and discussed in terms of the source distributions and strength. A limiting case, as the source strength is diminished and the stagnation point approaches the plane of the disk, is that of flow around a blunt-nosed half body.

Two integrable cases are discussed, namely, the case of disks of uniform source density, and disks with Bessel's distribution of sources. The methods of this paper should permit the solution of flow cases about new families of body shapes, many of which were not practicable before. An actual numerical solution of a given half body is not presented in the paper and would add to the value of the paper by presenting practical methods of calculation. V. L. Streeter, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 400, 493, 524, 525, 551, 560, 571, 575)

479. A. A. Townsend, "The measurement of double and triple correlation derivatives in isotropic turbulence," *Proc. Camb. phil. Soc.*, Oct. 1947, vol. 43, pp. 560–570.

This paper demonstrates how to use electrical analysis of the output from a hot-wire anemometer to make rapid and accurate measurements of quantities defining the decay of isotropic turbulence. The following circuits are exhibited in detail: (*a*) A single-stage circuit giving accurate time derivatives over a wide range of frequencies; (*b*) a circuit for measuring the mean product of the square of one voltage times a second voltage; (*c*) a circuit to measure the probability distribution, in time, of a fluctuating voltage.

Applications are made to: (1) The second and fourth derivatives of the longitudinal double-velocity correlation at the origin; (2) the third derivative of the longitudinal triple-velocity correlation at the origin; (3) the statistical distribution in time of the velocity fluctuations; and (4) the integral scale of turbulence.

Extensions of the method are indicated by an application to the integrals of the double-correlation curve.

Max G. Scherberg, USA

480. F. N. Frenkiel, "On the kinematics of turbulence," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 57–64.

This paper contains a brief review of theoretical and experimental data on the double-correlation coefficients and spectra in approximately isotropic turbulent flow. The author proposes special notations to distinguish 13 different double-correlation coefficients, measured at the same instant at two points and at the same point at two instants for different components of the fluctuations.

Much of the paper is concerned with the empirical representation of correlation curves and spectra, the formulas for correlation curves being of the type

$$R(r) = \exp(-k|r|)\Phi(r), \text{ or } \exp(-kr^2)\Phi(r).$$

Hugh L. Dryden, USA

481. H. W. Liepmann and J. Laufer, "Investigations of free turbulent mixing," *Nat. adv. Comm. Aero. Tech. Note No. 1257*, Aug. 1947, pp. 1–61.

The authors have made extensive measurements of the components of the local velocity in a two-dimensional free jet. The data have been analyzed with reference to their significance as criteria in testing the validity of the theories of turbulent mixing. The characteristic laws of jets, wakes, etc., are derived analytically, for the laminar case by the use of the integral equations of momentum and energy, and for the turbulent case with the added

aid of dimensional analysis. This shows that a hypothesis of the mechanism of turbulence is not necessary to the formulation of the phenomenological theories.

The data show that neither the mixing length nor the exchange coefficient is constant, and hence that theories based upon the assumption that these quantities are constant are not reliable. Previous verifications of such theories were possible because mean values of the variables were measured. The microscale of turbulence was found to be constant through most of the mixing zone, and von Kármán's fundamental relation between this and the scale of turbulence was verified. Extensive graphical presentation of the data is included.

The data in this paper should be compared with those being obtained independently by similar means at the Cavendish Laboratory. Dwight F. Gunder, USA

482. L. G. Loytsiansky, "Reciprocal action of the boundary layer on the distribution of pressure over the surface of a body in a flow" (in Russian), *Appl. Math. Mech.* (*Prikl. Mat. i Mekh.*) Mar.-Apr. 1947, vol. 11, pp. 205–214.

This paper presents a proof of the theorem that it is possible to construct a plane nonvortex flow of a perfect fluid (compressible in the general case) which is equivalent in distribution of pressure to a given actual flow. Prandtl was the first to mention the possibility of constructing such a flow, for the particular case of longitudinal flow of an incompressible fluid about a plate.

By applying this theorem the author obtains an expression for the resistance of a profile or a grid of profiles in terms of the "thickness of loss of impulse" in the aerodynamic wake, and the projection of the main vector of forces applied to a semibody or a grid of semibodies in a flow of a perfect fluid.

As another application the author gives a calculation of the heat per second obtained by transmission from the body and by dissipation of the energy in the boundary layer.

A bibliography is given, including reference to an earlier paper by the author. D. Jacovleff, Belgium

483. W. R. Sears, "The boundary layer of yawed cylinders," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 49–52.

An analytical study of the laminar boundary layer of an infinite yawed cylinder is presented. The study starts with the Navier-Stokes equations and Prandtl's argument regarding the order of magnitude of certain quantities. It is shown that the chordwise flow for the yawed cylinder is given by the same equations as for unyawed (plane) flow about the same cylinder. The study shows that the location of laminar separation of the chordwise flow is independent of yaw.

It is found that the boundary-layer flow over a flat plate, at zero incidence, is unaffected by the sweepback of the leading edge. Also, a certain class of cylinders suggested by Prandtl is considered in the yawed condition; the spanwise boundary-layer flow is calculated approximately and some characteristic features are presented graphically. R. C. Binder, USA

484. K. Viktorin, "Investigation of turbulent mixing processes," *Nat. adv. Comm. Aero. Tech. Memo. No. 1096*, Oct. 1946, pp. 1–32 (transl. from *Forsch. IngWes.*, 1941, vol. 12, no. 1).

Using water for both primary and secondary flow, the author measured the pressure and velocity distributions in the mixing region of a circular jet. The ratio of secondary to primary flow was varied from 1 to 4. Measurements were made with a type of Pitot-static tube.

In an attempt to provide a theory for the mixing region, the author adopts Prandtl's momentum-transport theory with mixing length constant over a transverse cross section. As is known, such theoretical velocity distributions can usually be brought into agreement with experiment, as shown by the work of Liepmann and Laufer [*Nat. adv. Comm. Aero. Tech. Note No. 1257*, 1947].

Finally, some comparisons are made between pressures in the mixing chamber, as measured and as calculated according to G. Flügel [*Forschungsh. Ver. dtsch. Ing.*, no. 395, 1939 (transl. in *Nat. adv. Comm. Aero. Tech. Memo. No. 982*, 1941)]. W. R. Sears, USA

485. Lester Lees, "The stability of the laminar boundary layer in a compressible fluid," *Nat. adv. Comm. Aero. Tech. Note No. 1360*, July 1947, pp. 1–144.

This study applies the methods of investigation of the stability of real incompressible flows of Heisenberg and Lin to the compressible-flow problem. Only subsonic disturbances are considered, because neutral "supersonic" disturbances would be arbitrary in wave length and phase velocity, and hence would have no significance for boundary-layer stability.

Basic general conclusions of interest are: (1) At large Reynolds numbers R, the influence of the viscous forces on the stability is essentially the same as in an incompressible fluid, (2) stability of an inviscid compressible flow (infinite Reynolds number) does not imply stability over the whole range of Reynolds numbers, (3) the viscous and inertia forces influence the stability of the laminar boundary layer in a compressible flow in such a way that for Reynolds numbers below some critical value viscosity dominates and stabilizes the flow, whereas at very high Reynolds numbers viscosity becomes destabilizing, (4) the effects of pressure gradients in compressible flows are of the same nature as in the case of incompressible flows.

Significant results in regard to thermal surface conditions are obtained from the following setting. In flows of low-level free-stream turbulence, the self-excited disturbance Reynolds number is a lower R boundary for transition, and its magnitude is substantially influenced by free-stream Mach number and thermal conditions on the surface.

Transition is delayed or advanced by flow of heat out of or into the surface, respectively. It is advanced on a neutral surface with increasing Mach number (the reviewer has observed this in studies of experimental data); it is permanently delayed at free-stream supersonic speed at some minimum-heat flow into the surface which depends only on the free-stream Mach number. It is estimated that for flight Mach speeds above an approximate value of 3, the radiation losses balance a critical rate of heat flow for boundary layer stability at all Reynolds numbers, provided adverse pressure gradients are not present.

The paper includes five appendixes on the calculation details, as well as tables and curves to facilitate the calculations.

M. G. Scherberg, USA

486. Hugh L. Dryden, "Some recent contributions to the study of transition and turbulent boundary layers," *Nat. adv. Comm. Aero. Tech. Note No. 1168*, Apr. 1947, pp. 1–44.

This paper summarizes the present state of knowledge of the turbulent boundary layer in two steps. It deals first with the stability of the laminar boundary layer and the problems of transition to turbulence.

The author discusses the mechanics of the turbulent boundary layer, investigating different theories in the light of his experimental results. The theoretical basis for understanding the instability of the laminar boundary layer was due to Tollmien, who showed that small disturbances of wave length in a certain

region would be amplified. No evidence was found to support Tollmien's theory until 1940, when the author and his co-workers discovered the existence of the Tollmien waves and these results were confirmed by H. W. Liepmann. The transition of the unstable waves to turbulence requires further experimental research. The author discusses the effect on this transition of free-stream turbulence, curvature, pressure gradient and suction.

Different existing theories concerning the fully developed turbulent boundary layer are mentioned in brief, among others those of Buri, Gruschwitz Kalikhman, Fediaevsky, and Nevzgljadov. The discussion and criticism of these theories based on systematic experimental investigation carried out under the writer's direction seem important, and the experimental method used for turbulent shearing stress measurement is interesting. One important conclusion indicated by these experiments is that the turbulence in the boundary layer is definitely nonisotropic. The semi-empirical theories are not quite satisfactory, according to the experiments. The author also points out fundamental objections to the Prandtl-von Kármán theories on the mixing length.

The experimental results seem to be in favor of von Kármán's theory concerning the statistical similarity of the fluctuations. The discrepancy between the direction of the principal axis of dilatation of the mean flow and that of the turbulent shearing stress is confirmed by these experiments on two-dimensional flow between two plates under pressure.

E. Abody-Anderlik, Hungary

487. Gabriel Viguier, "Equations of the boundary layer in the case of large-velocity gradients (Les équations de la couche limite dans le cas de gradients de vitesse élévés)," *C. R. Acad. Sci., Paris,* Mar. 10, 1947, vol. 224, pp. 713–714.

This brief note presents the differential equation of the boundary layer for plane flow of a viscous, incompressible fluid, when the velocity gradients are so large that the viscosity force cannot be taken as a linear function of the gradient. Instead, the viscosity force is assumed to be represented by a MacLaurin's series up to the third-power term. The resulting differential equation has all the terms of the linear case, and, in addition, a group of terms depending on the presence of a viscosity coefficient D involving the third power of the velocity. A dimensionless coefficient $T = \rho L^3/D U_o$ is introduced, in addition to the Reynolds' number $L U_o/\nu$.

Charles Concordia, USA

488. H. W. Liepmann and G. H. Fila, "Investigations of effects of surface temperature and single-roughness elements on boundary-layer transition," *Nat. adv. Comm. Aero. Tech. Note No. 1196,* Apr. 1947, pp. 1–31.

The laminar boundary layer and the position of the transition point on a heated flat plate are investigated. It is found that the Reynolds modulus of transition decreases as the temperature of the plate is increased, and that variable viscosity in the boundary layer produces a velocity profile with an inflection point, if the wall temperature is higher than the free-stream temperature. Studies of the flow in the wake of large, two-dimensional roughness elements are presented, and show that a boundary layer can separate and reattach itself to the wall without having transition take place.

Ahmed D. Kafadar, USA

489. A. M. Binnie and J. S. Fowler, "A study by a double-refraction method of the development of turbulence in a long circular tube," *Proc. roy. Soc., London, Ser. A,* Dec. 23, 1947, vol. 192, pp. 32–44.

With some refinements over the earlier work of one of the authors [*Proc. Camb. phil. Soc.,* 1941, vol. 37, p. 436 (with E. J. Bowen); and *Proc. phys. Soc. London,* 1945, vol. 57, p. 390] an experimental study has been made of the breakdown of developing laminar flow in a circular pipe. The experimental technique employs the double-refraction property exhibited by a very weak aqueous solution of benzopurpurin in a state of shear.

For two different entry conditions, the Reynolds number for the initial appearance of turbulent "flashes" is plotted against distance from the pipe entrance. The Reynolds number at which the fully developed turbulent flow is reached is plotted in a similar manner. The results agree satisfactorily with those of other investigators using different methods.

Although the apparatus arrangements are sketched, the hydrodynamic nature (e.g., turbulence levels and spectra) of the initial disturbances at the entrance is not given. Apparently no periodic laminar disturbance was observed prior to the onset of turbulence.

Stanley Corrsin, USA

Compressible Flow, Gas Dynamics

(See also Revs. 482, 485, 511, 520, 521, 522, 532, 536, 541, 549, 564)

490. Heybey, "Analytical treatment of normal condensation shock," *Nat. adv. Comm. Aero. Tech. Memo. No. 1174,* July 1947, pp. 1–28 (transl. from Heeres-Versuchsstelle, Peenemünde, Archiv. no. 66/72, 1942).

Neglecting the changes of mass flow and of the specific gas constants due to condensation, and considering only the heat liberated, the author examines normal condensation shocks in Laval nozzles, showing first that a stable shock can occur only at supersonic speed. Experiments show that, behind the shock, the velocity remains supersonic, speed and Mach number decrease, while pressure, density and temperature increase.

Theory alone cannot predict the Mach number for which the condensation shock occurs, nor its intensity, up to a certain maximum which is a function of the Mach number. If the Mach number at which the shock occurs and the heat liberated are known a complete analytical treatment is possible. The intensity of the shock (as measured by the pressure ratio) is a decreasing function of both the Mach number and of the heat liberated. If, for a given heat supply, the shock moves downstream, for a given section the Mach number and the velocity decrease, while pressure, density and temperature increase.

Leon Beskin, USA

491. R. Sauer, "Method of characteristics for three-dimensional axially symmetrical supersonic flows," *Nat. adv. Comm. Aero. Tech. Memo. No. 1133,* Jan. 1947, pp. 1–24 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin,* 1940, Res. Rep. no. 1269).

The method of characteristics for studying two-dimensional steady supersonic flows was developed by Prandtl and by Busemann ["Gasdynamik," Handb. der Experimentalphysik, Akadem. Verlag., Leipzig, 1931, vol. 4, p. 421]. The present paper was one of the earliest extensions of this method to flows around axially symmetrical bodies (see, however, Frankl's paper referred to below).

The author and others later extended the method to cover irrotational flows, and these later methods are now generally used in making three-dimensional calculations. In this connection reference may be made to later works by the author ["Theoretische einfuhrung in die gasdynamik," Springer, Berlin, 1943

(reprinted by J. W. Edwards, Ann Arbor, Mich., 1945, p. 137); Lilienthal Gesellsch. Luftfahrtforsch., Rep. no. 139 (2nd part translated as Gen. Elect. Rep. no. 71389)], by W. Tollmien [Wasserban Versuchsanstalt, Kochelsee, Arch. no. 44/4 and no. 44/6], by A. Vazsonyi [*Quart. appl. Math.*, Jan. 1948, vol. 4, p. 499], by G. Guderley ["Monograph on the theory of the characteristics," Air Materiel Command, T-2 report], as well as to the earlier work by F. Frankl ["Supersonic flows with axial symmetry," Izvestia Art. Akademii REKA, 1934, vol. 1 (transl. in Nav. Ord. Lab. Memo., no. 8538)].

The author transforms the hyperbolic equation for the velocity potential into the canonical form, in which the characteristics are the independent variables. This equation is interpreted as an expression which gives the new velocity components as one proceeds from a region in which the flow is given to a region in which the flow is desired to be known. Unlike the two-dimensional case in which the canonical equation contains only the velocities explicitly, the canonical equation in the axially symmetric flows involves the radial co-ordinate also.

The canonical differential equation is replaced by a difference equation which is used to construct a graphical network of corresponding points in both the flow field and the velocity field. Calculation of the successive points can be made using a slide rule; but a nomogram is given to aid some of the work. To obtain sufficient accuracy, it is necessary to use a method of iteration in which the nets are successively corrected by interpolation of the values obtained from preceding approximations.

The method is illustrated by applying it to the problem of flow in a conical nozzle and to a nozzle having parallel outflow. The analytical solution of the first case is known. For the second case a check is obtained from the condition of continuity. The agreement in both cases was quite good and three approximations appear to give acceptable convergence.

Francis H. Clauser, USA

**492. Henri Pailloux, "On the equations of motion of ideal fluids (Sur les équations du mouvement des fluides parfaits)," ***C. R. Acad. Sci., Paris*, Dec. 10, 1947, vol. 225, pp. 1122–1124.

The Euler equations of motion for a nonviscous compressible fluid are transformed by means of a simple identity, involving partial derivatives of products of the density and the linear velocity components. A second transformation involving products of the velocity components and the square root of the density results in a nearly symmetrical form for the differential equations.

Integral forms of the equations with respect to space and time are also indicated. M. J. Thompson, USA

493. Richard Scherrer, "The effects of aerodynamic heating and heat transfer on the surface temperature of a body of revolution in steady supersonic flight," *Nat. adv. Comm. Aero. Tech. Note No. 1300*, July 1947, pp. 1–27.

The author presents a method for determining the convective cooling requirements on the laminar boundary-layer region of a body of revolution in supersonic flight. His analysis is based on the fact that the rate of heat transfer by conduction, through or from a surface of unit area, by a surrounding fluid, is a function of the temperature gradient in the fluid and the thermal conductivity of the fluid at the surface. To determine the temperature gradient, the boundary-layer thickness must be calculated, which necessitates a knowledge of the temperature, velocity, Mach number, and air density of the fluid along the surface. This is obtained from a pressure distribution over the surface by the method of von Kármán and Moore. For higher Mach numbers or for blunt bodies, the pressure distribution may be obtained by the three-dimensional method of characteristics.

The boundary-layer thickness is calculated by using the general momentum equation for a laminar boundary layer on a body of revolution, with the simplifying assumptions: (1) The velocity profile in the boundary layer is linear for all Mach numbers and at all rates of heat transfer; (2) radiation effect is neglected; (3) the Prandtl number is taken as unity; (4) the ratio of specific heats is constant with changes in temperature.

The over-all rate of heat transfer or cooling requirement is obtained by integrating the local rate over the entire surface of the body. The author applies this method to a body with a fineness ratio of 8.8, with a completely laminar boundary layer, Mach numbers from 1.20 to 3.00, and at altitudes of 40,000 to 120,000 ft. The convective cooling requirements were found to be small but increased with Mach number.

Y. S. Touloukian, USA

494. Abe Gelbart, "On subsonic compressible flows by a method of correspondence—I. Methods for obtaining subsonic circulatory compressible flows about two-dimensional bodies," *Nat. adv. Comm. Aero. Tech. Note No. 1170*, Mar. 1947, pp. 1–35.

Assuming an adiabatic equation of state it is shown that the potential and stream functions φ, ψ satisfy differential equations of the type

$$\varphi_\theta = \tau_1(q)\psi_q, \ \varphi_q = -\tau_2(q)\psi_\theta$$

in the hodograph plane, where q, θ are magnitude and angle of velocity, and τ_1, τ_2 are positive analytic functions. These equations are satisfied by the real and imaginary parts of Σ-monogenic functions, which play a role similar to that of analytic functions in incompressible flow. The theory of Σ-monogenic functions is discussed briefly, and it is pointed out that the lack of a theorem analogous to that of Laurent in analytic functions is a major difficulty.

The Σ-monogenic functions defined by the so-called formal polynomials, Σ-monogenic Taylor's series and generalized Laplace integral represent solutions of the above equations. The formal transformation of such solutions from the hodograph to the physical plane is carried out. The compressible source and vortex, already found by Ringleb, are the only solutions so far obtained which represent flows of interest.

For incompressible flow φ and ψ satisfy the Cauchy-Riemann equations in the physical plane and in a distorted hodograph plane. Using the linearized equation of state, $p-p_1 \propto (1/\rho) - (1/\rho_1)$, it is shown that φ and ψ for compressible flow satisfy the Cauchy-Riemann equations in a distorted hodograph plane. Thus a complex potential for an incompressible flow may be used as the complex potential in the distorted hodograph plane for a compressible flow. The method is carried out in general form, yielding formulas for the complete solution of a compressible flow in terms of the complex incompressible potential function and an arbitrary analytic function. Lester L. Cronvich, USA

495. Shepard Bartnoff and Abe Gelbart, "On subsonic compressible flows by a method of correspondence—II. Application of methods to studies of flow with circulation about a circular cylinder," *Nat. adv. Comm. Aero. Tech. Note No. 1171*, Apr. 1947, pp. 1–39.

The report is an application of the method of correspondence presented in part I to flows in which the linearized equation of state is used. The formal solution of the general compressible-

flow problem is repeated and is shown to depend on a complex incompressible potential function and an arbitrary analytic function called the correspondence function. The correspondence function is given its most general form and the problem is reduced to the evaluation of unknown coefficients.

The problem of compressible flow with circulation around a circle is solved in detail. The complex potential function for the corresponding problem in incompressible flow is chosen, and the correspondence function is determined so as to yield a compressible-flow solution around a profile approximating a circle as closely as possible. It is pointed out that Tsien's solution [*J. aero. Sci.*, Aug. 1939, vol. 6, p. 399] reduces to a special selection of the correspondence function. Calculations are carried out for the approximate profiles (distorted unit circles) and corresponding velocity distributions for various angles of attack and subsonic Mach numbers. The distortion of the circle is shown to increase with Mach number. Lester L. Cronvich, USA

496. H. Eggink, "Compression shocks of detached flow," *Nat. adv. Comm. Aero. Tech. Memo. No. 1150*, June 1947, pp. 1–15 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1850, 1943).

The algebraic relations for oblique shocks are set up in a form which permits graphical solutions of the possible triple-shock intersections. The conditions for a solution are the usual ones, that downstream of the triple-shock intersection point the streamline is straight, with equal pressures and parallel velocities for the fluid that has passed through the single shock and the fluid that has passed through the two shocks. The nature of the solutions is discussed and several sample solutions are given in tabular form. To help in the construction of general solutions, a shock-polar chart is appended on which are drawn loci of the appropriate intermediate velocities.

The method of solution presented gives the interesting result that no triple-shock intersections occur below $(M_1^*)^2 = (\gamma + 1)\left(1 \pm \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right)$, using the negative sign. (This result is said to be incorrect in a cryptic note appended by the translator, which states in part that triple-shock intersections are "obviously" possible at all supersonic velocities; this is not obvious to the reviewer, nor is it obvious that the author's analysis is in error.)

The value of M_1^* using the positive sign in the above expression divides a region of one triple-shock intersection from that with two distinct such intersections.

Howard W. Emmons, USA

497. R. Finkelstein, "The normal reflection of shock waves," *Phys. Rev.*, Jan. 1, 1947, vol. 71, pp. 42–48.

This gives an approximate analytical integration of the hydrodynamical equations for a shock wave, perpendicularly incident on a perfect plane reflector. The complete pressure-time relation at the reflector surface is obtained. It is found that for gases the duration and magnitude of the impulse delivered to the reflector are greater than predicted by acoustic theory, whereas for liquids and solids they are less.

An equation of the form $(p + \pi)\, v^{\omega} = k$ is taken for the entire reflection process, shock included. Here p is gage or overpressure, v is specific volume, and π, ω, and k are constants for the given process. The pulse trailing the incident discontinuity is assumed to be progressive and of linear velocity distribution. Decay of the shock during the reflection process is neglected, and location of the discontinuity is thereby made possible without trial and error. This and further simplifications for the region between the discontinuity and the reflecting surface are justified by comparison with precise graphical integration of the shock and isentropic flow equations. The agreement is excellent, although somewhat forced by judicious interpretation of the known graphical results.

It appears that the present analysis will yield accurate results for shocks of pressure ratio $(p + \pi)/\pi \leq 1.6$, for which the acoustical theory would yield errors in total impulse of as much as ten per cent. H. G. Elrod, Jr., USA

498. W. Perl, "Calculation of compressible flows past aerodynamic shapes by use of the streamline curvature," *Nat. adv. Comm. Aero. Tech. Note No. 1328*, June 1947, pp. 1–88.

Calculations of compressible potential flow past simple aerodynamic shapes are made, based on the assumption of suitable streamline curvatures in the flow field. The results are expressed in terms of a local shape parameter $\sqrt{YC_a}$ (Y is the ordinate to a point on the surface and C_a is the curvature of the surface at that point) and a parameter η, with limiting values fixed by the boundary conditions. Velocity distributions about a Kaplan airfoil section, of thickness ratio 0.10, are compared to results obtained by other methods. The author considers satisfactory agreement to be achieved.

Applications of the curvature procedures to airfoils with mean camber line in the form of a circular arc, to bodies of revolution, and to supersonic flow are discussed briefly. A comparison of pressure coefficients is included for the standard compressibility correction methods. R. G. Folsom, USA

499. M. A. Heaslet, H. Lomax, and A. L. Jones, "Volterra's solution of the wave equation as applied to three-dimensional supersonic airfoil problems," *Nat. adv. Comm. Aero. Tech. Note No. 1412*, Sept. 1947, pp. 1–82.

The specialized form of Green's theorem, as used by Volterra for the solution of the two-dimensional wave equation, is directly applied to the linearized equation for the acceleration potential of three-dimensional supersonic flow. This brings a unification to the previous solutions obtained by various source or doublet distributions.

For this application, Volterra's method reduces to a surface integral which can directly yield solutions for flat-plate planforms and for uniformly loaded lifting surfaces.

This unification of the various solutions for three-dimensional wings in supersonic flow should prove very useful. However, the reviewer believes that the use of Hadamard's method (wherein he introduces the "finite part" of the integral to evaluate the integrals of Green's theorem on the characteristic surface or Mach cone where the integrand becomes infinite) is inherently a more powerful method of approach [A. G. Webster, "Partial differential equations of mathematical physics," G. E. Stechert & Co., New York, 1933, p. 277]. E. V. Laitone, USA

500. Max A. Heaslet and Harvard Lomax, "The use of source-sink and doublet distributions extended to the solution of arbitrary boundary value problems in supersonic flow," *Nat. adv. Comm. Aero. Tech. Note No. 1515*, Jan. 1948, pp. 1–48.

This note consists of a continuation of earlier work by the authors and A. L. Jones [*Nat. adv. Comm. Aero. Tech. Note No. 1412*, Sept. 1947]. These publications present restatements and applications of standard methods in partial differential equations to the linearized compressible-flow equations. The present note deals with the classic method of Hadamard and makes extensive

use of his concept of the finite part of a nonconvergent integral, without attempting any physical explanation of this concept. The mathematical treatments are perhaps unnecessarily lengthy.

The triangular or delta wing with lateral symmetry and with supersonic-type leading edges is investigated under rolling and pitching angular velocities, and analytic results for the pressure distribution are obtained. The stability derivative for damping in roll is developed. The stability derivative in pitch is not given in the paper but may be derived from the results given for the pressure. If the axis of rotation for the pitching motion is taken at the aerodynamic center ($^2/_3\, c$), no lift is produced and the damping moment may be expressed by

$$\frac{d\,C_m}{d\left(\frac{qc}{V}\right)} = -\frac{2}{9\sqrt{M^2-1}}$$

where the moment coefficient C_m is defined with respect to the total wing chord c, q is the free-stream dynamic pressure, V is the free-stream velocity, and M is the Mach number. It may be observed that the two stability derivatives are also valid for the reverse delta wing. Wallace D. Hayes, USA

501. Stewart Paterson, "Repeated reflection of shock waves (La réflexion répétée des ondes de choc)," *C. R. Acad. Sci., Paris*, Mar. 24, 1947, vol. 224, pp. 891–892.

The author shows that the results given by M. A. Herpin [*C. R. Acad. Sci., Paris*, 1946, vol. 223, p. 276] can be extended to the case of repeated shock waves in ideal gas. A perfect gas with a constant ratio of specific heats γ is in a tube having one end closed. A piston moves with a uniform velocity toward the closed end. By using Hugoniot's equation, the author derives relations giving ratios p_n/p_o and ρ_n/ρ_o as function of γ and n, where p_n, ζ_n are the pressure and density in the nth shock wave, p_o, ζ_o are those of the gas at rest. It is stated that details of this investigation will be published later.

Ahmed D. Kafadar, USA

502. Brown University, Grad. Div. Appl. Math., "Summaries of foreign and domestic reports on compressible flow, vols. I, II," *Hdqtrs. Air Mat. Comm.*, Wright Field, Dayton, Ohio, Tech. Rep. nos. F-TR-1168 (A and B)-ND, pp. 1–78, and 1–100, respectively.

The Graduate Division of Applied Mathematics is carrying out a project with the purpose of reviewing foreign and domestic work in the field of compressible flow and preparing an up-to-date treatise on it. The initial task was to screen and review the foreign (essentially German) documents.

The summaries presented here are concise analyses of selected documents on compressible flow and related material. They are intended to set forth the salient features of these papers so that the reader is informed of their contents and can determine whether or not he would be interested in securing them for further study. More volumes are to follow.

Gottfried Guderley, USA

503. Brown University, Grad. Div. Appl. Math., "Summaries of foreign and domestic reports on compressible flow, vol. III," *Hdqtrs. Air Mat. Comm.*, Wright Field, Dayton, Ohio, Tech. Rep. no. F-TR-1168C-ND, pp. 1–76.

The present volume is a continuation of vols. I and II of this series and contains summaries of 24 papers by various authors, and of the book "Mechanics of deformable media," by A. Sommerfeld. Almost all of the sources are German and the summaries are written by various members of the staff of Brown University. D. R. Mazkevich, USA

504. H. W. Sibert, "Approximations involved in the linear differential equation for compressible flow," *J. aero. Sci.*, Dec. 1947, vol. 14, pp. 680–681.

An attempt is made to investigate the order of the terms neglected in the usual linearization of the exact equation for two-dimensional motion of a perfect compressible fluid. It is concluded that the largest terms neglected are one order smaller (in the perturbation velocities) than those retained.

Allen E. Puckett, USA

505. W. E. Moeckel and J. F. Connors, "Charts for the determination of supersonic air flow against inclined planes and axially symmetric cones," *Nat. adv. Comm. Aero. Tech. Note No. 1373*, July 1947, pp. 1–35.

A set of charts is presented for convenient determination of flow conditions behind a shock wave at the surface of inclined planes and axially symmetric cones located in a uniform, frictionless, supersonic stream. Shock angle, static pressure coefficient, static pressure ratio, total pressure ratio, Mach number ratio, and velocity ratio for two-dimensional and conical flow are plotted, for Mach numbers ranging from 1.05 to infinity.

Ahmed D. Kafadar, USA

506. H. Bilharz and E. Hölder, "Calculation of the pressure distribution on bodies of revolution in the subsonic flow of a gas. Part I—Axially symmetrical flow," *Nat. adv. Comm. Aero. Tech. Memo. No. 1153*, July 1947, pp. 1–31 (transl. from *Dtsch. Luftfahrtforsch.*, Rep. no. 1169/1).

This report presents an adaptation of the classic method of von Kármán for incompressible flow to the subsonic region about a body of revolution, using the linearized theory.

Wallace D. Hayes, USA

507. B. Göthert, "Plane and three-dimensional flow at high subsonic speeds," *Nat. adv. Comm. Aero. Tech. Memo. No. 1105*, Oct. 1947, pp. 1–17 (transl. from *Lilienthal Gesellsch. Luftfahrtforsch.*, Rep. no. 127, p. 97).

The Prandtl-Glauert rule which states that the pressure coefficients of a thin two-dimensional object in a subsonic airstream vary as $1/\sqrt{1-M^2}$ is a well-known result of gas dynamics. A number of authors have incorrectly concluded that this same result applies to thin bodies of revolution. The author of this paper was, the reviewer believes, the first to give the correct method of extending the Prandtl-Glauert method to three-dimensional cases. His paper started an extensive discussion in the literature of the problem of treating three-dimensional flows, the final outcome being that the Göthert procedure is now accepted as being correct. See for example W. R. Sears [*Quart. appl. Math.*, July 1946, vol. 4, p. 191, and Apr. 1947, vol. 5, p. 89; *J. aero. Sci.*, Dec. 1946, vol. 13, p. 637], L. Lees [*J. aero. Sci.*, Dec. 1946, vol. 13, p. 638; *Nat. adv. Comm. Aero. Tech. Note No. 1127*], E. V. Laitone [*J. aero. Sci.*, Aug. 1946, vol. 13, p. 404, and Mar. 1947, vol. 14, p. 147; *Quart. appl. Math.*, July 1947, vol. 5, p. 227] and B. Göthert [Air Materiel Command T-2 Rep. F-TS-1514-RE, Apr. 1947].

The author applies his procedure to the problem of finding the effect of Mach number on the velocity at the surface and at a great distance from both two-dimensional wings and axially sym-

metric bodies. He gives the critical free-stream velocities for these cases. In addition he discusses the effect of Mach number on the lift of wings of finite aspect ratio. Later papers have revised the author's results for axially symmetric bodies to give a closer degree of approximation. Francis H. Clauser, USA

508. **T. von Kármán, "The similarity law of transonic flow,"** *J. Math. Phys.*, Oct. 1947, vol. 26, pp. 182–190.

Similarity parameters for slender bodies in transonic flow are developed by the author by means of perturbation analysis (around $u = a^*$, where u is the velocity component in the undisturbed flow direction and a^* is the critical sound velocity, or speed of sound where the local Mach number is unity) applied to the differential equations for the velocity potential in two-dimensional and in axially symmetric isentropic flows. A linear transformation is applied, which contracts the flow region as the free-stream Mach number M_1 approaches unity.

The resulting equation can be made independent of the thickness ratio τ, the ratio of specific heats γ, and M_1, by satisfying certain algebraic relations. For two-dimensional flow these relations lead to the similarity law

$$\frac{1 - M_1}{(\tau\Gamma)^{2/3}} = K = \text{constant}$$

where $\Gamma = {}^1/_2(\gamma + 1)$. The corresponding pressure, drag and lift coefficients are derived in functional form. The author concludes that at $M_1 = 1$ these coefficients are respectively proportional to the $^2/_3$, $^5/_3$ and $^2/_3$ powers of τ and to the $-{}^1/_3$ power of Γ. Under a certain assumption the pressure coefficient reduces to the Prandtl-Glauert rule for subsonic flow.

For axially symmetric flow the similarity law is

$$\frac{1 - M_1}{\tau^2\Gamma} = K_1 = \text{constant}$$

At $M_1 = 1$ it is predicted that the pressure and drag coefficients are proportional to τ^2.

The above similarity laws are shown to be of the form which will give the correct dependencies for the known exact solutions in slightly supersonic flow. It is pointed out that, since the theory applies to potential flow, it assumes that the effect of viscosity on shock-wave configuration and on flow separation is not highly variable in the range of Mach numbers considered.

Further work on this subject has been done by C. Kaplan [*Nat. adv. Comm. Aero. Tech. Note No. 1527*, Jan. 1948].

Arnold M. Kuethe, USA

Aerodynamics of Flight; Wind Resistance

(See also Revs. 493, 498, 499, 500, 505, 506, 508, 537, 538, 542, 546, 548, 559, 561)

509. **J. Weissinger, "The lift distribution of sweptback wings,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1120*, Mar. 1947, pp. 1–38 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1553, 1942).

A lifting-surface method (designated F) and a lifting-line method (designated L) for calculating the spanwise lift distribution on a rectangular wing are described. For the F method, the down-wash at any point of the surface is expressed as the sum of two integrals, of which one is the familiar expression for the down-wash due to the trailing vortex system given by the Prandtl lifting-line theory, and the other embodies the corrections due to the surface distribution of bound vorticity. Two assumptions are then made: (*a*) the chordwise distribution of vorticity is taken to be the same as that on an infinite flat plate; (*b*) it is assumed that the spanwise distribution is determined with adequate accuracy by equating the total down-wash to the incidence along the three-quarter chord line only.

It is shown that the numerical solution of the resulting integral-differential equation can be obtained on lines analogous to Multhopp's method for solving the Prandtl lifting-line equation. The method is applied to wings of aspect ratio ranging from 1 to 6, and comparison with the results given by the Prandtl lifting-line method shows the latter to be increasingly in error with decrease of aspect ratio. The extension of the method to swept-back wings is only briefly indicated, but results of spanwise lift distribution are given for an aspect ratio of 5 and sweepback angles of 0, 15 and 45 deg.

The L method assumes the loading to be concentrated on the quarter chord line, and the local incidence is made equal to the down-wash at the three-quarter chord line. Again the resulting equation is amenable to numerical solution on lines analogous to the Multhopp method.

The F and L methods are found to give results in good agreement, the latter being less laborious. Comparison with the results of the lifting-line method developed by Multhopp for sweptback wings shows the latter to be appreciably in error for large sweepback angles. Comparison with available experimental results is not conclusive. A. D. Young, England

510. **A. W. Goldstein and M. Jerison, "Isolated and cascade airfoils with prescribed velocity distribution,"** *Nat. adv. Comm. Aero. Tech. Note No. 1308*, May 1947, pp. 1–50.

A number of exact methods are available for calculating the pressure distribution about arbitrary cascades of airfoils for incompressible two-dimensional flow, and the inverse problem of finding the airfoil contour to produce a prescribed pressure distribution. They are cumbersome, and sufficiently precise approximate methods have been devised by several authors for practical use. The present paper describes a method of treating the inverse problem, which is approximate in the sense that only the contour on the low-pressure side of the airfoil is accurately determined, the assumption being made that this is the critical side from the standpoint of shock formation or flow separation.

The basis of the method is to replace the airfoil boundary by vortices. An approximate airfoil shape is assumed and the vortex distribution along the boundary thus determined. The stream function along the boundary is then computed. The shape is successively adjusted until the stream function becomes constant along the airfoil boundary, indicating that the boundary is the correct one for the assumed pressure distribution. If the initially assumed airfoil shape differs appreciably from the correct one, the process of shape adjustment by this method is found to converge rather slowly. In a typical case illustrated in the report, seven successive approximations were required.

John V. Becker, USA

511. **H. R. Ivey, G. W. Stickle, and Alberta Schuettler, "Charts for determining the characteristics of sharp-nose airfoils in two-dimensional flow at supersonic speeds,"** *Nat. adv. Comm. Aero. Tech. Note No. 1143*, Jan. 1947, pp. 1–37.

This paper presents graphs and tables of the following flow characteristics on the two sides of two-dimensional shock or compression waves, for flow-incidence angles up to 70 deg, and incident Mach speeds up to 9: (*a*) Flow-deflection angles; (*b*) up and downstream Mach speeds; (*c*) ratios of up and downstream pressures; and (*d*) pressure coefficient drops. Comparable data are

given for expansion waves. Such data enlarged to include total head ratios, density ratios, mass-flow ratios, etc., can be very useful labor-saving tools. Max G. Scherberg, USA

512. J. Mayo Greenberg, "Airfoil in sinusoidal motion in a pulsating stream," *Nat. adv. Comm. Aero. Tech. Note No. 1326*, June 1947, pp. 1–20.

This paper deals with the analytical determination of the force, moment, and wake form for a harmonically oscillating airfoil in a harmonically pulsating stream. The problem is restricted to the two-dimensional incompressible case of a flat-plate airfoil, for which suitable velocity potentials can be determined. The results are considered applicable to the case of a rotating-wing aircraft blade with a periodic variation in angle of attack.

• M. J. Thompson, USA

513. Arthur L. Jones, Mildred G. Flanagan, and Loma Sluder, "An application of lifting-surface theory to the prediction of angle-of-attack hinge-moment parameters for aspect-ratio 4.5 wings," *Nat. adv. Comm. Aero. Tech. Note No. 1420*, Sept. 1947, pp. 1–16.

A graphical method for solution of the direct problem of the finite lifting surface was given by D. Cohen in 1942. This permits a correction to hinge-moment parameters calculated by lifting-line theory.

In the present paper slight modifications (not completely specified) of the Cohen method are made. Down-wash velocities are calculated for an elliptical planform of aspect ratio 4.5, and compared with those interpolated from results of Crandall and Stewart [*Nat. adv. Comm. Aero. Tech. Note No. 1175*, 1947] for elliptical planforms of aspect ratios 3 and 6. The slope of the hinge-moment vs. angle-of-attack curve is computed for four horizontal tail configurations, and compared with experimental results and calculated slopes given by Crandall and Stewart. Agreement is good. W. H. Pell, USA

514. L. Agnelluzzi, "Center of drift (Centro di deriva)," *Riv. aero.*, Sept. 1947, vol. 23, pp. 507–515.

Asymmetric motions and aerodynamic forces are studied. A metacenter of drift is defined as the location in the plane of symmetry of a plane surface having an effect equivalent to that of the various components of the airplane in a relative lateral wind.

The author theorizes that for a turn effected with ailerons alone, this metacenter should be behind and below the center of gravity of the airplane, but for a turn effected with rudder only, it should be behind and above the center of gravity.

The primary effect of the area and location of the vertical tail surface, as well as the effect of the fuselage, wing, undercarriage, and propeller on the location of the metacenter are studied with a view to arriving at a satisfactory position. An analysis for determining the position of the metacenter from wind-tunnel tests is also given, and confirmatory flight tests reported.

R. Contini, USA

515. F. Ringleb, "Some aerodynamic relations for an airfoil in oblique flow," *Nat. adv. Comm. Aero. Tech. Memo. No. 1158*, June 1947, pp. 1–15 (transl. from *Dtsch. Luftf.*, Rep. no. 1497, 1941).

In this report certain aerodynamic relations are derived for "straight" and "oblique" flow, the former being defined as a flow perpendicular to the leading edge of an airfoil, the latter as a nonperpendicular flow. The fluid is assumed to be incompressible, and the velocity to be uniform and constant at infinity. Two cases are considered: (1) the airfoils are identical for straight and oblique flow, so that the profiles in the direction of flow are different, and (2) the airfoils are not identical, but the profiles in the direction of flow are the same.

Formulas expressing the lift and moment coefficients for oblique flow as functions of the corresponding coefficients for straight flow are derived for each case. The author assumes that the reader is familiar with the method of "conformal mean" which he had previously developed, but (apparently through error or misprint) gives no specific reference for an explanation of the method. A few other aerodynamic relations are also given, including the rate of change of lift coefficient with angle of attack, which has been experimentally verified.

C. W. Smith, USA

516. A. Fage, "Aerodynamic research at the National Physical Laboratory" (in English), *Ingenieur, 's Grav.*, Dec. 12, 1947, vol. 59, pp. L 81–90.

This paper is a brief résumé. Topics discussed include performance of aircraft wings, research equipment and technique, airplane flutter, and airplane control and stability. No references to papers on the original work are given. J. M. Wild, USA

517. H. G. Lemme, "Force and pressure-distribution measurements on a rectangular wing with a slotted droop nose and with either plain and split flaps in combination or a slotted flap," *Nat. adv. Comm. Aero. Tech. Memo. No. 1108*, Mar. 1947, pp. 1–11 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1676/2, 1943).

The author reports experiments made on a wing containing a slotted droop-nose installation in combination with flaps. The slot between the leading edge of the wing and the droop nose was introduced on the basis of previous tests, which indicated separation due to the low-pressure region occurring at the hinge point of a plain, unslotted droop nose. The slot proved ineffective in the configuration tested, and revealed no advantages over the simple droop nose. Andrew Fejer, USA

518. A. P. Webster, "Free falls and parachute descents in the standard atmosphere," *Nat. adv. Comm. Aero. Tech. Note No. 1315*, June 1947, pp. 1–30.

This paper presents complete calculations, tables, and nomograms for free falls and parachute descents. The calculations are based on assumptions of equilibrium between drag and weight, and constant-drag coefficient.

Excellent agreement is shown between the calculations and observed jumps, including free falls, from altitudes of as high as 40,000 ft. Robert T. Jones, USA

519. Charles F. Hall, "The effect of modifications to the horizontal tail profile on the high-speed longitudinal control of a pursuit airplane," *Nat. adv. Comm. Aero. Tech. Note No. 1302*, Aug. 1947, pp. 1–69.

In this study of the effect of wind-tunnel modifications to the horizontal tail profile on longitudinal stability and control, two symmetrical stabilizers (a modified NACA four-digit and an NACA 65-series airfoil), two flat-sided elevators, and three elevators with bulged profiles were tested. The tests covered Mach numbers from about 0.30 to 0.80. Data on pressure distributions over the tails is given.

The results of the tests show: (1) Except for small changes in the elevator angle for zero pitching moment, the profile changes have almost no effect on pitching moment characteristics of the

model; (2) The effect of a bulge on the elevator profile, with the stabilizers tested, is to change the quantities $\partial C_{h_e}/\partial \delta_e$ and $\partial C_{h_e}/\partial C_L$ (where C_L and C_{h_e} are the lift and elevator hinge-moment coefficients and δ_e is the elevator angle) from negative values to smaller negative or positive values, and to cause the elevator angle corresponding to zero hinge moment to become more negative with increase in Mach number; (3) With either stabilizer, the flat-sided elevators produce a severe reversal in the variation of stick force with air speed at high speeds. A small bulge on the elevator with the low-drag stabilizer reduces the severity of the reversal appreciably, but does not increase the speed at which it occurs. A large bulge on the elevator with either stabilizer eliminates the reversal throughout the wind-tunnel test range; (4) The effect of the bulged elevator profile is to reduce the stick-force gradient. Alexander Klemin, USA

520. Harold B. Pierce, "Tests of a 45-deg sweptback-wing model in the Langley gust tunnel," *Nat. adv. Comm. Aero. Tech. Note No. 1528*, Feb. 1948, pp. 1–25.

Comparative tests on a 45-deg sweptback wing and an equivalent straight wing were made in the Langley gust tunnel to obtain data for the prediction of gust loads on high-speed wings.

These tests and calculations indicate that the gust loads on sweptback-wing airplanes are dependent on the lift-curve slope of the equivalent straight wing multiplied by the cosine of the angle of sweepback, and on the effect of the gradual penetration of the gust on the unsteady lift function. Results also indicate that the maximum acceleration increment of an airplane with a sweptback wing would be less than that for the same airplane with an equivalent straight wing. H. R. Gillespie, Jr., USA

521. H. R. Ivey and E. B. Klunker, "Considerations of the total drag of supersonic airfoil sections," *Nat. adv. Comm. Aero. Tech. Note No. 1371*, July 1947, pp. 1–16.

Results of calculations of the viscous and pressure drags of some two-dimensional supersonic airfoils at zero lift are presented. These results indicate that inclusion of viscous drag alters many previous conclusions regarding the desirability of certain airfoil shapes for securing low drags at supersonic speeds.

Drag calculations for six per cent thick symmetrical circular-arc and double-wedge airfoils are presented for Mach numbers of 1.35 and 1.6, and Reynolds numbers from 10^5 to 10^8.

Ahmed D. Kafadar, USA

522. B. Göthert, "High-speed measurements on a sweptback wing (sweepback angle φ = 35 deg)," *Nat. adv. Comm. Aero. Tech. Memo. No. 1102*, Mar. 1947, pp. 1–33 (transl. from *Lilienthal Gesellsch. Luftfahrtforsch.*, Rep. no. 156, p. 30).

The curves of lift, moment, and drag for a sweptback wing have been determined up to a Mach number of 0.87, and are compared to those of a rectangular wing.

Through measurements of the total head loss behind the wing, and through schlieren pictures, an insight into the formation of the compression shock at high Mach numbers has been obtained.

Ahmed D. Kafadar, USA

523. J. G. Lowry and S. M. Crandall, "Wind-tunnel investigation of unshielded horn balances on a horizontal tail surface," *Nat. adv. Comm. Aero. Tech. Note No. 1377*, July 1947, pp. 1–36.

Results of a wind-tunnel investigation to determine the aerodynamic characteristics of a horizontal tail surface with various amounts of unshielded horn balance are presented in this paper. It is indicated that the variations of hinge-moment coefficient with angle of attack and with elevator deflection are approximately linear functions of the ratio of horn-area moment to elevator-area moment. Lift, drag, and pitching moment coefficients are also presented as functions of angle of attack and elevator deflection. John E. Goldberg, USA

524. N. V. Lambine, "Flow with separation along an isolated profile of almost rectilinear form (Le mouvement d'un courant avec détachement suivant un profil isolé de forme à peu près rectiligne)," *C. R. Acad. Sci. URSS*, Feb. 28, 1947, vol. 55, no. 6, pp. 481–483.

The problem treated by the author in this paper was suggested by a study of L. Posduine on the theory of propellers operating under conditions of flow separation. It deals with potential flow about slightly curved profiles of zero thickness at angles of attack associated with flow separation at the leading and trailing edges. For these conditions the author investigates, theoretically, profile shapes which give a maximum lift-drag ratio at any given angle of attack. The problem is approached by application of a conformal transformation of Levi-Civitta. Three cases are worked out numerically. Results show that a concave curvature of the profile increases the lift-drag ratio. Frank L. Wattendorf, USA

525. K. Bussmann and A. Ulrich, "Systematic investigations of the influence of the shape of the profile upon the position of the transition point," *Nat. adv. Comm. Aero. Tech. Memo. No. 1185*, Oct. 1947, pp. 1–51 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin, Jb. dtsch. Luftfahrtforsch.*, 1943, p. 1).

The Schlichting-Ulrich method [*Jb. dtsch. Luftfahrtforsch.*, 1942, p. 18] is used to calculate the initial instability point for the set of Joukovsky profiles obtained by varying the thickness from 0 to 25 per cent of the chord in 5 per cent steps, and the camber from 0 to 8 per cent in 2 per cent steps. Lift coefficients of 0.00, 0.25, 0.50, and 1.00, and Reynolds numbers of 10^4, 10^5, 10^6, 10^7, and 10^8 are used. The results are presented in an extensive series of graphs.

These results should be considered as only qualitative in nature in view of: (1) the uncertainty which exists regarding the relation between the instability point and what may properly be called the transition point, and (2) the recent discovery of a numerical error in the original Schlichting calculations.

William Pell, USA

526. H. A. Lemme, "Force and pressure-distribution measurements on a rectangular wing with double-hinged nose," *Nat. adv. Comm. Aero. Tech. Memo. No. 1117*, Mar. 1947, pp. 1–6 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1676/3, 1944).

This report presents a continuation of the author's efforts to improve the value of maximum lift coefficient of an airfoil by bending the nose downward, thereby increasing the camber and resulting lifting force of the airfoil.

Previous tests with a single-hinged nose showed a large low-pressure peak at the bend which favored flow separation. To decrease this effect, the amount of camber was reduced by adding another section while maintaining the same total deflection angle of the most forward section from the chord line. This produced no increase in the maximum lift coefficient nor in the value of the negative pressure peak, however. It is concluded that the greatest gain in maximum lift coefficient (from 0.72 to 1.30) can be achieved by bending a single nose section, of length 11 per cent

(or perhaps even slightly less) of the chord, through an angle of about 35 deg. The tests were made along the center section of an end-plated NACA 0009-E4 airfoil, at a Reynolds number of about 1,000,000. E. Arthur Bonney, USA

527. W. R. Bates, "Collection and analysis of wind-tunnel data on the characteristics of isolated tail surfaces with and without end plates," *Nat. adv. Comm. Aero. Tech. Note No. 1291*, May 1947, pp. 1–177.

This presents low-speed wind-tunnel data on the aerodynamic characteristics of a variety of tail surfaces, of various planforms, with and without end plates, and with sealed and open gaps. The data is plotted in terms of various aerodynamic and geometric parameters and may be of value in arriving at optimum design of stabilizer-elevator units. Analytical results are also presented and compared with the experimental data. These indicate fair verification of the lifting-surface-theory procedure for obtaining aerodynamic coefficients for tail surfaces.

John E. Goldberg, USA

528. Elie Carafoli and Nicolas Tipei, "Aerodynamic characteristics of rectangular wings swept back with parabolic axis (Sur les caractéristiques aérodynamiques des ailes rectangulaires en dérive et à axe parabolique)," *C. R. Acad. Sci., Paris*, Dec. 10, 1947, vol. 225, pp. 1121–1122.

This brief note gives the results of theoretical calculations of the down-wash angle for wings of finite span, and of rectangular planform with a sweepback.

The case of the wing with straight sweepback is considered, as well as that where the spanwise axis of the wing is in the form of a parabolic arc. M. J. Thompson, USA

529. P. E. Purser, M. L. Spearman, and W. R. Bates, "Preliminary investigation at low speed of down-wash characteristics of small-scale sweptback wings," *Nat. adv. Comm. Aero. Tech. Note No. 1378*, July 1947, pp. 1–55.

This is a collection and brief analysis of down-wash measurements made at low speeds behind small-scale sweptback wings of various aspect ratios, sweepback angles and ratios of root chord to tip chord, with and without leading-edge slots and trailing-edge flaps. The aerodynamic characteristics of the various wing-tail combinations are plotted, and the effect of the wing configuration on the rate of change of down-wash angle with angle of attack is analyzed. J. S. Isenberg, USA

530. R. M. Crane and R. W. Holtzclaw, "Wind-tunnel investigation of the effects of profile modification and tabs on the characteristics of ailerons on a low-drag foil," *Nat. adv. Comm. Aero. Rep. No. 803*, 1944, pp. 1–50 (publ. in 1947).

The purpose of the tests reported in this paper was to obtain quantitative data on the effects of aileron profile and trailing-edge modifications and the effects of tabs on the characteristics of ailerons on a low-drag airfoil, and to form a logical basis for the specification of aileron tolerances. Previous experiments have indicated that thickening and beveling the control-surface trailing edge is a powerful means of adjusting hinge-moment characteristics. Typical conclusions obtained are as follows:

"Deviations of the order of ±0.005 of the aileron chord from the specified profile on the ailerons of a typical pursuit airplane can cause stick-force variations of ±20 lb for a large rate of roll at an indicated airspeed of 300 mph. It is also shown that the danger of overbalance at small deflections of closely balanced ailerons can be diminished by thickening of the aileron profile if the internal-balance chord is simultaneously reduced, to maintain the same stick force for a large rate of roll.

"Thickening and beveling the trailing edge on a typical aileron installation caused a reduction of 50 per cent in the control force for a large rate of roll at high speed. When used in conjunction with internal balance, the thickened and beveled profile resulted in a 30 per cent reduction in the nose balance required for a given control force at high speed. Under these conditions, the variation of control force with rate of roll was more nearly linear for the aileron of normal profile than for the ailerons with thickened and beveled trailing edges."

Basic data are presented from which the effect of tabs can be calculated for specific cases. The data are sufficient for the solution of problems of fixed tabs with a differential linkage, as well as simple and spring-linked balancing tabs.

In addition, a great number of graphs are given of test data showing the quantitative effects of contour and thickness on lift-curve slope, section-drag coefficient, hinge moments, stick force, rate of roll, and the pressure across an aileron nose-balance seal for many different conditions of contour and thickness.

E. Arthur Bonney, USA

531. Laurence K. Loftin, Jr., "Theoretical and experimental data for a number of NACA 6A-series airfoil sections," *Nat. adv. Comm. Aero. Tech. Note No. 1368*, July 1947, pp. 1–43.

This gives theoretical pressure-distribution data and ordinates for NACA 6A-series airfoils, with a thickness range from 6 to 15 per cent, and with locations of minimum pressure at from 30 to 50 per cent of chord. While retaining relatively high critical Mach numbers, this new series of airfoils eliminates the thin trailing edge which is characteristic of the earlier 6-series, thus reducing design and fabrication difficulties.

Two-dimensional wind-tunnel tests are also described, which indicate that minimum-drag and maximum-lift characteristics of the 6 and 6A-series are essentially the same, while the pitching moment coefficient for the 6A-series is slightly more negative.

John E. Goldberg, USA

532. L. A. Galin, "Notes on the theory of a wing of finite span in a supersonic flow" (in Russian), *Appl. Math. Mech. (Prikl. Mat. i Mekh.)*, May-June 1947, vol. 11, pp. 383–386.

The author considers a supersonic flow around an airfoil of finite span. If we assume that the motion takes place in the x-direction, then the potential function φ satisfies the simplified compressibility equation $\varphi_{zz} + \varphi_{yy} - (M^2 - 1)\, \varphi_{xx} = 0$ and the boundary conditions $\partial\varphi/\partial z = 0$ for $z = 0$ in front of the wing (inside the Mach cone). For the value of z belonging to the intersection S of the wing (and lying inside the Mach cone) with the x, y-plane, we have $\partial\varphi/\partial z = f(x, y)$, and $f(x, y) = \partial F(x, y)/\partial x$, where $z = \pm F(x, y)$ is the equation of the wing.

Using known results on representation of solutions of the wave equation, the author obtains an approximate solution in the form $\Psi(x, y, z) =$

$$\frac{1}{\pi} \int_S \int f(\xi, \eta) \left[(x/\sqrt{M^2 - 1} - \xi)^2 - (y - \eta)^2 - z^2 \right]^{-1/2} d\xi\, d\eta$$

The problem of the vibrating wing is treated in a similar manner. In this case the potential is assumed in the form $\Psi(x, y, z) + Re\Phi(x, y, z)e^{i\beta x - \delta\omega t}$, where Φ satisfies the equation $\Phi_{yy} + \Phi_{zz} - \Phi_{xx} - \lambda^2\Phi = 0$, and λ is a constant. Using Hadamard's formulas for the potential of a simple layer for the above equation, the

author obtains an approximate formula satisfying the derived boundary conditions. Stefan Bergman, USA

533. **John R. Hagerman, "Wind-tunnel investigation of the effect of power and flaps on the static lateral stability and control characteristics of a single-engine high-wing airplane model,"** *Nat. adv. Comm. Aero. Tech. Note No. 1379,* July 1947, pp. 1–74.

The author presents experimental data, obtained from wind-tunnel tests, of a model tested with flaps undeflected and with single-slotted and double-slotted flaps deflected, with the tail surfaces both installed and removed, and with varied power conditions.

In general the results indicate that, with the flaps neutral, the application of power does not change the effective dihedral of the airplane appreciably, whereas the application of power decreases the effective dihedral with either of the two flaps tested deflected. The directional stability of the airplane was increased by the application of power at all flap deflections. Six component force and moment measurements in yaw are reported, for various configurations. Rudder effectiveness and hinge-moment data are included. W. F. Milliken, Jr., USA

534. **G. Thiel and F. Weissinger, "Six-component measurements on a straight and a 35-deg sweptback trapezoidal wing with and without split flap,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1107,* June 1947, pp. 1–61 (transl. from *Dtsch. Luftfahrtforsch.*, Res. Rep. no. 1278).

Tests were made through the positive range of angles of attack, and for yaw angles up to ± 30 deg. The predicted and experimental variations in aerodynamic coefficients and stability derivatives were in good agreement for the unstalled wings. The results near or above the stall are of limited value because of the low Reynolds number(1×10^6). Philip Donely, USA

535. **F. H. Imlay, "Theoretical motions of hydrofoil systems,"** *Nat. adv. Comm. Aero. Tech. Note No. 1285,* June 1947, pp. 1–78.

Starting with the fundamental equations for force and moment, an exploratory theoretical study was made of the longitudinal and lateral stability characteristics of hydrofoils in three arbitrary arrangements. The aerodynamic characteristics of the seaplane lifting surfaces and the virtual mass of the hydrofoils were not included in the analysis.

The resulting equations were not linear, and the necessary approximations for a solution corresponded with those usually applied in the theory of small oscillations. Limiting values of parameters for longitudinal stability of the hydrofoil arrangements were obtained. The approximate results with respect to lateral stability have very limited value. No suitable experimental data were available for an evaluation of the analytical methods. Typical dimensions selected corresponded to hydrofoil systems suitable for model tests. R. G. Folsom, USA

536. **Robert M. Snow, "Aerodynamics of thin quadrilateral wings at supersonic speeds,"** *Quart. appl. Math.*, Jan. 1948, vol. 5, pp. 417–428.

This paper presents a study of supersonic aerodynamics in some special cases of quadrilateral wing planforms. The study is limited to plane, infinitely thin wings at small angles of attack, since it uses the linearized supersonic-flow theory based on Busemann's method of conical fields. The method is applicable to swept, trapezoidal, and general quadrilateral wing planforms. Wing dihedral is also considered. Effects of wing thickness, viscosity, and interference are outside the scope of the method.
H. P. Liepman, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 393, 401, 532)

537. **J. Mayo Greenberg, "Some considerations on an airfoil in an oscillating stream,"** *Nat. adv. Comm. Aero. Tech. Note No. 1372,* Aug. 1947, pp. 1–47.

The velocity potential, lift force, moment, and propulsive force on a two-dimensional airfoil in a stream of periodically varying angle of attack have been derived, using nonstationary incompressible potential-flow theory including the effect of the continuously shed sheet of vortices from the trailing edge.

The results are applied to the propulsive force of an airfoil in an oscillating stream, and to the forced vibrations of an airfoil, including the airfoil stiffness and torsional axis location. Results of the theory are believed to be applicable to counterrotating propellers and to lifting surfaces operating in an oscillating stream.
H. P. Liepman, USA

538. **Th. Schade and K. Krienes, "The oscillating circular airfoil on the basis of potential theory,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1098,* Feb. 1947, pp. 1–38 (transl. from *Luftfahrtforsch.*, 1940, vol. 17, and 1942, vol. 19).

This is an extension of work of W. Kinner in 1936 to an oscillating circular airfoil disk in a uniform flow. The following motions of the disk are considered: (1) Flapping, in which the amplitude of displacements perpendicular to the uniform flow (assumed to be in the plane of the disk) is constant throughout the disk; (2) torsional oscillations about a central y axis in the plane of the disk, normal to the direction x of the undisturbed flow; (3) flexural oscillations with parabolic deformations, in which the amplitude is proportional to x^2 and to y^2; (4) torsional (rolling) oscillations about the x axis; (5) bending oscillations in which the amplitude is proportional to xy.

Both a velocity potential Φ and a pressure (or acceleration) potential φ are employed in the analysis. The latter, which has been used by Prandtl, is defined by $\bar{b} = \text{grad } \varphi$, where $\bar{b}$ is the acceleration vector, and can be shown to be proportional to the pressure at the upper and lower surface of the airfoil disk. For each of the above basic motions, the pressure potential is considered to be composed of functions of the "first kind" and of the "second kind." Those of the first kind are obtained from the Laplace equation $\Delta\varphi = 0$ in elliptic co-ordinates, and correspond to a down-wash velocity of the form $W_1 = f(x, y)e^{i\nu t} + g(y)e^{i(\nu t - \omega x)}$, where $f(x, y)$ denotes the function appropriate to the particular type of motion considered [for instance $f(x, y) = C\,xy$ for (5) above]. The pressure potential of the second kind is obtained by certain differentiations of that of the first kind, and corresponds to a down-wash of the form $W_n = C_n h_n(y)e^{i(\nu t - \omega x)}$. The condition is imposed that $W_{\text{tot}} = W_1 \sum_n W_n = f(x, y)e^{i\nu t}$. In the course of the analysis it is found that, as usual, the condition of smooth flowoff from the trailing edge must be applied.

The analysis leads finally to a system of linear equations, the coefficients of which can be calculated exactly with the aid of exponential functions and Hankel's functions. Numerical results are shown by a series of tables and graphs which give, for each of the five basic motions, values of the real and imaginary parts of

the lift and moment coefficients for a range of values of the reduced frequency $\nu c/V$ (c = radius of the disk) from 0 to 2.0.

Morris Morduchow, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 397, 401, 404, 423, 471, 482, 510, 512, 554, 555)

539. Hunt Davis, "A new method for the aerodynamic design of multistage axial-flow compressors," *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 41–48.

This paper is devoted to a discussion of inlet Mach number, and to the description of a graphical method for the determination of blade angles for an axial-flow compressor of free vortex design. A method is described for a graphical design utilizing the same rotor-blade shape for several successive stages.

Charles Concordia, USA

540. A. Kahane, "Charts of pressure rise obtainable with airfoil-type axial-flow cooling fans," *Nat. adv. Comm. Aero. Tech. Note No. 1199*, Mar. 1947, pp. 1–55.

Especially at altitude it is a problem to get the proper air-pressure rise for engine cooling from an axial-flow fan. The use of highly cambered 65-series blower-blade sections instead of conventional low-cambered sections has made higher pressures possible.

Charts and examples are presented for single-stage fans to show performance at propeller speeds and with gear drive. The effects of initial air-stream rotation are included. This report is comprehensive, but it still may be said that for actual design of an axial-flow fan the method of Curt Keller ["Axial flow fans," McGraw-Hill Book Co., Inc., 1937] is somewhat more straightforward.

William C. Johnson, Jr., USA

541. B. Pochobradsky, "Effect of centrifugal force in axial-flow turbines," *Engineering*, Mar. 21, 1947, vol. 163, pp. 205–207.

Conditions are established for constant mass-flow rate per unit area at the exit of a turbine nozzle diaphragm, assuming small losses but taking into account the pressure gradient due to centrifugal force and the compressibility of the fluid. Examples of the required radial variation of exit-flow angle are given for specific cases. It is shown that free vortex flow (axial-component constant and whirl component inversely proportional to radius) cannot be produced without a radial shift of the streamlines, since the mass-flow rate per unit area varies with radius.

W. D. Rannie, USA

542. Yoshinori Shimoyama, "Tests of cascades of airfoils for retarded flow," *Nat. adv. Comm. Aero. Tech. Memo. No. 1190*, Oct. 1947, pp. 1–28 (transl. from *Trans. Soc. mech. Engrs.*, Japan, 1937, vol. 3, p. 334).

This paper discusses the results of systematic tests on a low-speed compressor cascade of Gottingen 549 profiles. The author determines from pressure distributions the effect of spacing, vane angle and angle of attack on the lift and drag, and compares his results with Weining's well-known theoretical solution.

He shows that with decreasing spacing and vane angle, the zero-lift angle, slope of the lift curve, and drag increase, while the maximum lift decreases.

Weining's flat-plate lattice results show that the flow direction at the outlet approaches the direction of the vanes and that the interference between vanes depends on spacing and vane angle alone. It has frequently been suggested that these results might be applied to a cascade of airfoils having camber and thickness, by replacing the profiles by flat plates set at the zero-lift direction of the profiles. However, these tests indicate that such an application is not entirely justified, since the zero-lift direction of profiles in a lattice is affected by the lattice parameters, and since the interference depends not only on spacing and blade angle but also on the angle of attack.

Andrew Fejer, USA

543. R. P. Coleman and A. M. Feingold, "Theory of ground vibrations of a two-blade helicopter rotor on anisotropic flexible supports," *Nat. adv. Comm. Aero. Tech. Note No. 1184*, Jan. 1947, pp. 1–59.

This note is an extension of previous work on the theory of self-excited mechanical oscillations of hinged rotor blades, to cover the missing case of one or two blades on supports having unequal stiffnesses in two principal directions. The mathematical analysis involves the consideration of differential equations with periodic coefficients. Because the methods employed may be useful in other problems, the analysis is presented in considerable detail.

The results show the existence of ranges of rotational speed at which instability occurs, similar to those characteristic of the two-blade rotor on symmetric supports, but changed somewhat in position and extent. In addition, there is shown to be an infinite number of unstable regions occurring at low speeds. All of these unstable regions are found without considering aerodynamic effects.

The analysis is somewhat simplified in the special cases of infinite or zero stiffness in one of the support axes, and it is considered that designers may find these cases adequate, in conjunction with the results which have been previously presented. A design chart giving the location of the principal instability range for the case of infinite support stiffness in one axis is included. This case is of particular interest since it is equivalent to the case of a counterrotating-rotor system of two equal rotors revolving at equal and opposite speeds within the same support.

The effect of damping was not considered in the mathematical analysis. However, the authors believe that the unstable regions occurring at low speeds may be controlled by a small amount of damping, and that sufficient damping in both the supports and the blade hinges may also eliminate trouble from the primary self-excited vibration instability.

Charles Concordia, USA

544. C. F. Toms, "The performance of rotating-wing aircraft rotors," *Aircr. Engng.*, May 1947, vol. 19, pp. 150–158, 162.

This paper considers chiefly the basic principles of autogyros and helicopters, using a system of force and torque coefficients and parameters similar to those customarily used for propellers. In its most general form the resulting theory could be applied graphically to blades of any shape and twist, but the analytical applications given in the paper are restricted to: (1) constant chord and zero twist; (2) chord varying inversely with radius and zero twist; and (3) chord varying inversely with radius and with such twist as is required for uniform lift distribution.

There is a good qualitative discussion of rotor flow and blade forces, as well as of general operating principles. A number of charts and equations are given from which design and performance data can be obtained for the cases considered. Except for the method of analysis, the paper probably contains nothing essentially new, but is a useful compilation of theory and basic data which should be of value in this field.

E. G. Allen, USA

545. **E. Pollmann, "Temperatures and stresses on hollow blades for gas turbines,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1183*, Sept. 1947, pp. 1–108 (transl. from *Zentrale f. wissensch. Berichtswesen, Berlin*, Res. Rep. no. 1879, 1943).

This paper deals with calculations and tests on temperatures and stresses in hollow blades for gas turbines with great thoroughness. The paragraph on temperature variation and stress along the blade with heat conduction to the rotor disk is a distinct contribution. The effect of radiation is also considered. Some interesting test results on blade models are reported, in which the heat-transfer coefficient is tested by electrically heating the blade. The testing and analysis of temperature measurements on thin trailing-blade edges is interesting.

Unfortunately the reading of this paper is rather difficult and a more careful editing would be of great value to the reader. Numerous misprints occur in the text and in the formulas throughout the paper. H. E. Sheets, USA

Experimental Flow Equipment and Technique

(**See also Revs. 489, 516, 522**)

546. **Bertram J. Eisenstadt, "Boundary-induced up-wash for yawed and sweptback wings in closed circular wind tunnels,"** *Nat. adv. Comm. Aero. Tech. Note No. 1265*, May 1947, pp. 1–76.

Wind-tunnel testing of yawed and swept airfoils has received increased interest because of maneuvers involving large angles of attack and high speeds. The corresponding tunnel corrections are difficult since the problem cannot be reduced to that of two-dimensional flow, as with a straight airfoil. For rectangular tunnels a solution is possible by the method of images.

For the circular tunnel a method is described in which the tunnel-induced potential is broken up into two parts, that of a reflection vortex system which makes the tunnel a streamline far from the airfoil, and a residual potential whose effect is zero at infinity. The tunnel-induced velocities for yawed and sweptback airfoils in a closed circular wind tunnel are determined. The calculations are made for elemental horseshoe vortices having one tip of the bound vortex on the tunnel axis, for a range of yaw angles and bound-vortex lengths. The correction for complete yawed and sweptback wings of arbitrary span loading is obtained by a superposition of solutions. Charts and tables of the induced velocity normal to the plane of the tunnel axis and bound vortex are presented.

Formulas are also given for the tunnel-induced velocity normal to any other plane containing the tunnel axis. These are needed for sweptback wings at high angles of attack, where the tunnel axis and the two halves of the wing do not all lie in the same plane. Curves are presented for converting the tunnel-induced velocities into corrections to the geometric angle of attack of the wing. Alexander Klemin, USA

547. **A. E. von Doenhoff and F. T. Abbott, Jr., "The Langley two-dimensional low-turbulence pressure tunnel,"** *Nat. adv. Comm. Aero. Tech. Note No. 1283*, May 1947, pp. 1–66.

The author gives a complete but concise description of the Langley two-dimensional low-turbulence pressure tunnel and of its full-scale, atmospheric pressure equivalent. Particular emphasis is placed on the necessity for and the problems of attaining a uniquely low stream turbulence.

The wall-reaction method for measurement of lift is discussed, and some comparisons are made between lift force determined with this device and with conventional balance or the use of pressure-distribution integrations. Theoretical wall-interference factors are considered in the light of experimental results, and some interesting test equipment (particularly the liquid "integrating" manometer) is described.

H. Julian Allen, USA

548. **R. D. Munnikhuysen, "High-range true air-speed indicator,"** *ATI Tech. Data Dig.*, Mar. 1, 1948, vol. 13, pp. 7–12.

This paper gives a brief description and analysis of a new instrument for measuring true air speed throughout the subsonic, transonic, and supersonic regions. Thus far only a laboratory unit has been built, consisting basically of: (1) a controlled spark gap together with a spark source; (2) an ion detector, located a fixed distance downstream from the spark gap; and (3) a synchroscope or speed indicator which registers on a cathode-ray screen the time interval required by the vehicle to travel a distance through air equal to the distance from spark gap to ion detector. John E. Goldberg, USA

549. **J. H. Weaver, "A method of wind-tunnel testing through the transonic range,"** *J. aero. Sci.*, Jan. 1948, vol. 15, pp. 28–34.

This paper outlines a method of modifying existing high-speed wind-tunnel test sections with a suitably contoured "bump" on which reflection-plane (half) models can be satisfactorily tested through the speed of sound. The method depends on the establishment of a local high-velocity region over the bump, in which half models mounted vertically from its surface may be tested.

The ordinates of the bump contour which was found to give the best results are given, and it is shown that, for this contour, the Mach number decreases almost linearly no more than $\Delta M = 0.03$ in a distance of 8 in. from the surface of the bump. The balance system is discussed and shown diagrammatically.

In addition, experimental data on the lift, drag, and pitching moment of the P-80, on a typical fuselage and tail, and the effect of sweepback on the transonic drag characteristics of a high-speed family of wings, are given for the range of $M = 0.7$ through 1.1, as obtained by the above described method.

E. Arthur Bonney, USA

550. **Alb. Schlag and André Jorissen, "Contribution to the normalization of Venturi tubes (Contribution à la normalisation des tubes de Venturi),"** *Rev. gen. Hyd.*, Jan.-Feb. 1947, vol. 13, pp. 4–11, and Mar.-Apr. 1947, vol. 13, pp. 69–78.

In two closely correlated installments, this paper reviews the known characteristics of the Venturi meter and presents systematic data designed to extend such knowledge. Attention is focused upon the "classical" meter, of the form devised by Clemens Herschel, and the "standard" meter, consisting of the European standardized nozzle with the addition of a diffuser cone, as adopted by the International Federation of the National Standardizing Associations.

Extensive comparisons are drawn between the contributions of Italian, German, French, and English investigators and the original data of the authors, with regard to the effect of the Reynolds number, the meter proportions, the piezometer locations, and the roughness of the boundary. A selected bibliography is given. Hunter Rouse, USA

551. **R. A. Runyan and R. J. Jeffries, "Empirical method for frequency compensation of the hot-wire anemometer,"** *Nat. adv. Comm. Aero. Tech. Note No. 1331*, June 1947, pp. 1–24.

An extremely convenient method for the setting of proper

amplifier compensation (to correct for hot-wire time lag) is presented. Two fixed-frequency signals (100 and 500 cps) are fed into the hot-wire bridge, when the hot-wire is at operating condition in the turbulent flow. Two sharply tuned detector amplifiers transmit these signals from the regular amplifier output into a balance meter, which is brought to zero by adjustment of the resistance in the R-C compensation circuit. L. S. G. Kovasznay [*Nat. adv. Comm. Aero. Tech. Memo. No. 1130,* 1947 (from Müegyetem Aerodinamikai Intézetében Készult Munka, Budapest, 1943)] has used a similar technique using a square wave.

In reference to the more standard aspects of the equipment, it should be noted that the design frequency range of the amplifier (10 to 1000 cps) is too narrow for accurate study of many turbulent flows, particularly at high velocities. A broadening of the frequency range to values covered by standard hot-wire equipment should be accompanied by a shift of compensator range to lower values of the time constant.

Stanley Corrsin, USA

Hydraulics; Transport of Solids; Cavitation

(See also Revs. 398, 399, 400, 403, 477, 550, 558)

552. Giulio Quarisa, "Backwater curves in prismatic channels (Profili di rigurgito in alvei prismatici)," *Energia elett.,* Aug. 1947, vol. 24, pp. 328–339; and Sept. 1947, vol. 24, pp. 374–382.

The problem of backwater curves in prismatic channels was first dealt with by Bresse in 1860 when he considered the case of large rectangular channels. One of the latest attempts to solve this problem on a more general basis is the paper of P. de Varennes e Mendonca ["Backwater curves (Curvas de regolfo)," Lisbon, 1945].

The solution presented by the author involves writing the general equations of flow in open channels in the form

$$\frac{ix}{h_o} = \phi_1 + A\,\phi_2$$

where i is the slope of the prismatic channel, h_o the water depth for uniform flow, A a constant, and ϕ_1 and ϕ_2 are functions of the ratio h/h_o alone, where h is the depth at point x.

The writer develops this method and gives comprehensive calculations and diagrams for several forms of prismatic channels, including channels with trapezoidal and triangular cross sections.

Charles Jaeger, England

553. I. Léviant, "Study of varying flow in an open canal (Étude sur les écoulements variés en canal découvert)," *Rev. gen. Hyd.,* Mar.-Apr. 1947, vol. 13, pp. 59–68; July-Aug. 1947, vol. 13, pp. 198–201.

The paper refers to the varying flow in an open channel (steady flow with varying depth and varying velocity along the channel), and deals principally with the hydraulic jump. The author writes the well-known equations of energy (Bernoulli) and momentum (Euler) in relative values. He makes the assumption that the discharge Q in the underlying water jet below the vortex is constant, and that no water penetrates from outside into the rotating vortex overlying the jet. He then calculates the energy of the water jet and finds that this energy always exceeds the energy of a free flow with the same depth and the same discharge.

Introducing the equation of momentum, he finds a relation between the depth h of the underlying jet and the total depth H of the flow, including the vortex covering the jet. The equation is $h' = 2/(2M'_1 - H'^2)$, where $h' = h/h_c$, $H' = H/h_c$, $h_c = \sqrt[3]{Q^2/(gb^2)}$ (critical depth) $M' = M/(bh_c^2)$, $M = (bh^2/2) + Q^2/(gbh)$ = momentum, and b = width of canal. The index "1" refers to the upstream end of the jump, where $h'_1 = H'_1$.

Analyzing the flow of the water in the hydraulic jump, the author came to the conclusion that it should be possible to realize the passage from shooting flow to streaming flow without a jump and without internal losses, by giving a peculiar shape to the bed of the canal. This was tried, and such a flow was realized in a rectangular canal with a certain shape of the bed. The internal losses were reduced from 3.35 cm for a horizontal bed to only 1.18 cm, which nearly realizes the theoretical flow without internal losses. It is assumed that this new type of flow will have many practical applications.

Charles Jaeger, England

554. A. C. Merrington and E. G. Richardson, "The breakup of liquid jets," *Proc. phys. Soc. London,* Jan. 1947, vol. 59, pp. 1–13.

This paper describes experiments on the breakup of liquid jets, in which the diameter d of the resulting drops is measured. Both fixed and moving nozzles were used.

At high velocities the jets appear to be disrupted; the diameter of the drops is independent of the surface tension, and is proportional to $\nu^{0.2}/V$ where ν is the kinematic viscosity and V is the relative velocity between the jet and the surrounding air. At low velocities there are "varicose" disturbances of the jets, which break into drops having a size of the order of the diameter of the nozzle. The critical velocity found for the change-over from the first type of breakup to the second is compared with that given by Ohnesorge in 1936 and found to be quite different for very viscous liquids. Measurements made on the length of the jets confirm the work of Tyler and Richardson in 1925, but also show a discrepancy for very viscous liquids.

Factors influencing the stability of the drops when the nozzles are very large are briefly discussed in connection with the work of Bond and Newton in 1928. The experimental results in this part of the paper appear to be only qualitative.

Gino Moretti, Italy

555. H. H. Anderson, "Efficiency and cavitation of fluid machines," *Proc. Instn. mech. Engrs.,* 1947, vol. 157, pp. 85–87.

It is customary to assume that friction losses in hydraulic machines vary as the square of the speed, and to show deviations from this assumption by plotting the head-loss factor against the Reynolds number. Losses in water turbines can be estimated by Moody's empirical formula, and the cavitation losses in centrifugal pumps can be expressed as functions of the Thoma parameter. But both losses can be regarded as equivalent to the loss in a certain length of pipe and plotted against the Reynolds number.

The author has analyzed the performance of several series of pumps of the multistage, single-entry type with guide passages, having specific speeds, (rpm) $\sqrt{\text{gpm}}/(\text{ft head})^{3/4}$, in the neighborhood of 1000 to 1200. The pump efficiencies η were plotted against Q/D (where Q is the quantity of flow, D the diameter), which is proportional to Reynolds number if the kinematic viscosity is regarded as constant. It was found that the efficiency loss $(1 - \eta)$ could be represented as equivalent to the loss in a pipe whose length is 35 diameters. This method of analysis facilitates extrapolation, since $(1 - \eta)$ for a given pump can be found from a pipe friction curve plotted against Q/D. A correction for variations in viscosity can be made by a method given in the paper.

Cavitation causes a suction head or total head loss. As such

it could be expressed as a percentage of the total theoretical head and analyzed as an efficiency loss, but this would limit the analysis to geometrically similar designs. For more general analysis, the author plots his experimental data as suction-specific-speed against the Reynolds number equivalent Q/D_e (D_e is the diameter of the impeller eye). The mean curve agrees with a portion of the Reynolds friction-loss curve. The author states that extrapolation on this curve is more accurate than when using Thoma's parameter, or the use of suction-specific-speed as the sole determinant of cavitation, as suggested by Wislicenus. Karl E. Schoenherr, USA

Marine Propulsion

(See Rev. 535)

Lubrication; Wear

556. **P. I. Orloff, "Coefficient of friction, oil flow and heat balance of a full-journal bearing,"** *Nat. adv. Comm. Aero. Tech. Memo. No. 1165*, Oct. 1947, pp. 1–56 (transl. from *Aero. Engrg. Moscow*, 1935, vol. 9, p. 25).

The topics enumerated in the title are discussed from both an analytical and an experimental point of view. The procedure followed is to discuss the experimental findings of other workers and then proceed to fit a simplified theory to these results.

In the discussion of the coefficient of friction, the well-known Petroff analysis is considered first, followed by a treatment of the loaded journal bearing. In the latter analysis, the total friction is resolved into two components, that due to shear-induced flow and that due to pressure-induced flow. The results of the friction analysis are used to establish the inadequacy of the commonly used PV factor as a measure of the thermal stress of a bearing.

The component of the flow of lubricant through a bearing due to the pressure developed within the bearing is considered, as well as that due to forced feed. The several factors involved in making a thermal balance for a journal bearing are enumerated and an example of the procedure to be followed is presented. The full-floating bearing and the use of increased clearance in the unloaded region of a bearing are briefly discussed as means of lowering the operating temperature. Milton C. Shaw, USA

557. **J. T. Burwell, "The calculated performance of dynamically loaded sleeve bearings,"** *J. appl. Mech.*, Sept. 1947, vol. 14, pp. 231–245.

The author discusses the general equations of dynamically loaded film-lubricated sleeve bearings derived by Harrison and Swift for no leakage. He analyzes special cases such as constant steady, constant rotating, and periodic alternating loads of various frequencies, including the case of a nonrotating shaft. He develops a method based on these equations, by which any dynamically loaded bearing with known load-time relation can be analyzed. He also shows that the solution of such a problem can be applied to other bearings for which the shape of the load-time curve remains unchanged.

His reasoning is: Periodic variations in the magnitude and direction of the load cause the journal center to describe a path. The parametric equations of this path are obtained from the general equation, expressing η (the eccentricity ratio, e/c) and φ (the angle between load line and line of centers, measured in the direction of rotation from the minimum film-thickness end) as functions of time. Even the initial application of a steady load causes the journal center to move on closed orbits about the equilibrium position determined from Sommerfeld's theory.

The *maximum eccentricity ratio* η_{max} obtained from this path determines the *minimum film thickness* and hence the safe limits of bearing operation. *The load-carrying capacity* on the other hand is influenced by the *radial velocity* $d\eta/dt$ and the *angular velocity* $d\varphi/dt$ *of the journal center.*

The reviewer considers it important that the squeeze-film component, determined by $d\eta/dt$, is in some cases responsible for a higher peak load being carried by dynamically loaded bearings than by statically loaded bearings of the same eccentricity. This has also been discussed by A. G. M. Michell ["The mechanical properties of fluids," Blackie and Son Ltd., London, 1936, p. 119] and by D. D. Fuller [*Mach. Design*, Sept. 1947, p. 127].

To simplify the general equations the author introduces an eccentricity function $\zeta = \eta/(1-\eta^2)^{1/2}$, and uses a differential analyzer to find a closed-path solution (ζ versus φ) for the journal center.

Using a leakage factor K = (load capacity of actual bearing)/(load capacity of infinitely long bearing) which varies with η, the author suggests the following approximate method of considering leakage: After finding the no-leakage closed-path solution, a K versus ζ curve is plotted using published data. Since for $\eta > 0.58$, the load varies linearly with ζ, by changing every value of ζ to a corrected ζ_f, using the relation $(a\,\zeta + b)/K = \zeta_f + b$, the expected path ($\zeta_f$ versus φ) of the journal center can be obtained. Nicholas Sag, Australia

Dynamics of Meteorology and Oceanography

(See also Revs. 402, 567)

558. **J. J. Dronkers, "Methods for the calculation of tides (Methoden van getijberekening)"** (in Dutch with English summary), *Ingenieur, 's Grav.*, Dec. 5, 1947, vol. 59, pp. B121–137.

The author first discusses the remarkable studies of tides made in Holland, especially the work of the Commission for Drainage of the Zuider Zee. These investigations of the propagation of tides in bays, and in the lower sections of water currents receiving supply from upstream, are based on work by Lorentz and his collaborators, particularly the author and Mazure. The author distinguishes between an exact method, in which the friction term proportional to U^2 is retained and a second "linear" method in which this term is replaced by one proportional to U.

In the first section, the author reviews the meaning of the various types of tides indicated by M_2, M_4, etc., and specifies the characteristics of the various problems. In the second section he writes down the two differential equations

$$\frac{\partial h'}{\partial x} = \frac{1}{bhg}\frac{\partial s}{\partial t} + \frac{s|s|}{C^2b^2h^3} - \frac{(b+B)}{g\,b^2h^2}\,s\,\frac{\partial h}{\partial t}$$

$$\frac{\partial s}{\partial x} = B\frac{\partial h'}{\partial t}$$

where x = distance along the channel, positive upstream; t = time; s = volume per second of flow, positive for the ebb current; b = width of the channel; h = depth of the water; C = Eytelwein constant; $b + B$ = width of the channel at high tide; h' = height of water surface above a horizontal plane.

The exact solution presented is a series solution in which the various terms are calculated by successive approximations. The solution is given as a function of $h(O, t)$ and of $s(O, t)$, representing respectively variations of the height of the water and of the flow

rate in the downstream section as functions of time. In general, $s(O,t)$ is not given, and the author presents various means of using boundary conditions for solving the various problems.

In the third section, the author gives a solution by the linear method which involves only functions of real variables, whereas the method of Mazure used functions of complex variables. The author starts out from the above differential equations and transforms them, as Mazure did, by means of Fourier series. After deriving his solution he applies it for tides M_2 and M_4 to: (*a*) a channel closed on the upstream side; (*b*) a channel with supply from upstream; and (*c*) a system of branching channels.

To get the most out of this paper, it is advisable to review the previous papers by the author and Mazure.

L. J. Tison, Belgium

559. William Lewis, "A flight investigation of the meteorological conditions conducive to the formation of ice on airplanes," *Nat. adv. Comm. Aero. Tech. Note No. 1393*, Aug. 1947, pp. 1–54.

This is a report of the meteorological results of airplane flight tests made during the winter of 1945–1946 to determine the conditions contributing to the formation of ice.

The author describes the apparatus used for measuring the temperature of the air, the liquid water content, the mean diameter and size distribution of the drops, and the concentration of ice particles.

The results are given in a series of diagrams, but the most interesting part of the paper is the discussion of how the results may be interpreted for predicting the conditions which lead to formation of ice. More precisely the problem treated is that of estimating in advance the rate of ice formation in a cloud of given type, dimensions, and temperature.

The results obtained permit the author to present a series of laws which indicate the rate of ice formation for different types of clouds (stratus and cumulus), under various conditions.

L. J. Tison, Belgium

560. D. R. Davies, "Turbulence and diffusion in the lower atmosphere with particular reference to the lateral effect," *Proc. roy. Soc., London, Ser. A*, July 8, 1947, vol. 190, pp. 232–244.

In this paper the author extends the theory of atmospheric turbulence put forward by O. G. Sutton in 1934 by considering the lateral effects, which are important near bounding edges and have led to a discrepancy with experimental results for finite evaporating surfaces.

The author assumes that the evaporating saturated-liquid surface is a horizontal parabolic area with its axis in the mean wind direction (taken as the x direction), the vertex pointing upwind. He neglects the transfer of vapor due to the diffusion in the x direction of the mean wind, and derives a differential equation for the vapor concentration which contains, besides the usual term due to diffusion in the vertical z direction, a new term for transfer of vapor across y = constant planes. Defining the coefficient of diffusivity in the y direction in the same way as is usually done for the z direction, the author first solves the differential equation of diffusion under the assumption that the wind and both diffusivities are constant with height above the evaporating surface.

Using this as a first step he then goes over to the general case, with the wind and diffusivities variable with the height. Assuming that the wind obeys the z^m law used by O. G. and W. G. L. Sutton in their work on evaporation, with similar laws for the coefficients of diffusivity, the author obtains the expression for the vapor concentration, as well as for the rate of evaporation at a point on the saturated area, and finally the total rate of evaporation from the parabolic area. These new theoretical results are compared with measurements made in a wind tunnel, and a striking improvement of the agreement is found.

At the end of this important paper other possible extensions of the present theory are discussed.

Z. Sekera, USA

561. C. N. Warfield, "Tentative tables for the properties of the upper atmosphere," *Nat. adv. Comm. Aero. Tech. Note No. 1200*, Jan. 1947, pp. 1–56.

Upon the recommendation of the NACA Special Subcommittee on the Upper Atmosphere, tentative tables for the properties of the upper atmosphere have been prepared for the region from 20 to 120 km (65,000 to 393,000 ft).

Two sets of tables are presented, the first of which is based upon the same arbitrary constant value for the acceleration of gravity that was used in the preparation of existing tables for the lower levels. The second set is based upon an inverse square relation between acceleration of gravity and altitude. A table of latitude correction factors is also included for still greater precision. Quantities listed in the tables as a function of altitude include temperature, pressure, density, specific weight, coefficient of viscosity, speed of sound, and mean free path of the air molecules. For heights above 80 km each table is divided into parts for day and night conditions.

John E. Goldberg, USA

Ballistics; Detonics (Explosions)

(See also Rev. 493)

562. E. Roth and R. Sänger, "Critical remarks on the method of determination of the ballistic density of air according to S. Dufrênois and O. von Eberhard (Kritische Betrachtungen über die Verfahren von S. Dufrênois und O. von Eberhard zur Bestimmung der ballistischen Luftdichte)," *Schweiz. Arch.*, Jan. 1948, vol. 14, pp. 22–32.

About 25 years ago, S. Dufrênois and O. von Eberhard each developed an approximate method for determining the ballistic air density. The purpose of this paper is to compare these two methods and to determine their accuracy. To achieve this aim the authors determine the ballistic air density directly from the ballistic equations, by the perturbation methods of K. Schwarzschild and P. Stanke.

It is shown that in a number of typical cases the approximations of Dufrênois and von Eberhard are in excellent agreement with the results obtained by perturbation methods. For the particular illustrations chosen by the authors it seems that the procedure of Dufrênois gives somewhat better corrections for horizontal range and time of flight than does the von Eberhard procedure. However, the differences are probably small in comparison with the dispersions due to variations from round to round.

Benjamin Epstein, USA

563. D. C. Pack, W. M. Evans, and H. J. James, "The propagation of shock waves in steel and lead," *Proc. phys. Soc. London*, Jan. 1948, vol. 60, pp. 1–8.

The stress system set up by an explosive detonating in contact with a metal surface is investigated. From an extrapolation of Bridgman's data on the compressibility of steel and lead, it is concluded that the initial velocity of the shock wave generated by the detonation is less in steel and greater in lead than the velocity of plane elastic waves in these media (that is the general "velocity of sound" for waves of dilation, involving neither shear nor rotation).

Measurements have been made of the time taken by the fastest pulse to traverse various lengths of steel and lead. The results confirm that the velocity of propagation of plane elastic waves is greater than the initial shock-wave velocity in steel. In lead, the shock wave has an initial velocity well in excess of that of the elastic waves, decaying after some distance to an elastic wave.

Stuart R. Brinkley, Jr., USA

564. G. N. Abramovich and L. A. Vulis, "On the mechanics of the propagation of detonation and burning" (in English), *C. R. Acad. Sci. URSS*, Jan. 20, 1947, vol. 55, no. 2, pp. 107–110.

In this paper a study is made of the propagation of detonation, assuming that it consists of a normal shock wave followed by a burning wave. With this picture, formulas are derived analogous to Prandtl's well-known shock-wave relation $M_1^*M_2^* = 1$, where $M^* = V/a^*$ = fluid velocity divided by the critical velocity. By using the equations of continuity and momentum for the burning wave, which follows immediately behind the shock wave and converts the stream M_2^* into stream M_3^*, the authors derive the relation

$$\frac{T_{03}}{T_{02}} = \frac{M_3^{*2}(M_2^{*2}+1)^2}{M_2^{*2}(M_3^{*2}+1)^2}$$

where T_{03} is the stagnation temperature after the detonation wave, and T_{02} the stagnation temperature after the shock wave but before the burning wave. From this formula is obtained the following relation between M_3^* and M_2^*

$$M_3^* = \frac{1+M_2^{*2}}{2M_2^*}\left[1-\sqrt{1-\frac{4M_2^{*2}}{(1+M_2^{*2})^2}\,\frac{T_{03}}{T_{01}}}\right]\sqrt{\frac{T_{01}}{T_{03}}}$$

On making the approximations that M_2^* is generally considerably below unity and that the ratio of stagnation temperature is approximately 1, the following simple formula is derived, giving the appropriate approximate generalization of Prandtl's equation for a detonation wave

$$M_3^* \cong \frac{1}{M_1^*}\sqrt{1+\theta\left(1-\frac{\gamma-1}{\gamma+1}M_1^{*2}\right)}$$

where $\theta = Q/(c_pT_1)$ is the thermal characteristic of the mixture. On analyzing this relation it is shown that the detonation wave has the well-known characteristic (usually obtained by tedious computations) that there is a minimum permissible velocity at which M_3^* is unity.

It is noted that at the opposite extreme the detonation-wave velocity is identical to the shock-wave velocity for the given initial and final states, for infinite propagation velocity. A formula is derived relating the propagation velocity M_1^* to the thermal characteristic of the mixture θ, which simplifies in practical cases to

$$M_1^{*2} = \frac{2+4\theta}{1+4\,\dfrac{\gamma-1}{\gamma+1}\,\theta}$$

This formula is said to give values in error by less than two per cent from the more exact formula, which is also derived.

Howard W. Emmons, USA

565. H. R. Ivey, E. N. Bowen, Jr., and L. F. Oborny, "Introduction to the problem of rocket-powered aircraft performance," *Nat. adv. Comm. Aero. Tech. Note No. 1401*, Dec. 1947, pp. 1–46.

This paper is a useful compilation of the basic concepts of rocket-powered flight. The authors discuss the equations for zero-lift flight of a rocket, with emphasis on the fundamental limitations on performance.

Expressions are derived for the range, altitude, rocket velocity, and jet velocity as functions of the propellant and the rocket weight, and of the nozzle dimensions. The results are clearly discussed and plotted so as to promote ease of manipulation.

P. Rosenberg, USA

Thermodynamics

(See also Revs. 490, 540, 560, 564)

566. W. C. Johnston, "Measures flame velocity of fuels at low pressures," *Soc. auto. Engrs. J.*, Dec. 1947, vol. 55, pp. 62–65.

Experimental flame-velocity measurements using the Bunsen burner technique were made on natural gas, acetylene, and gasoline, at Reynolds numbers below 2000.

It was found that at pressures not above atmospheric, flame velocity increases with inlet temperature, and the flame velocity for acetylene increases with decrease in pressure. The range of air-fuel ratio for stable burning increases with inlet temperature and pressure. The data is presented in curve form, and diagrammatic sketches are given of the apparatus used.

M. G. Scherberg, USA

567. N. Fuchs, "Concerning the velocity of evaporation of small droplets in a gas atmosphere," *Nat. adv. Comm. Aero. Tech. Memo. No. 1160*, Aug. 1947, pp. 1–20 (transl. from *Phys. Z. Sowjet.*, 1934, vol. 6, p. 224).

A discussion is given of the rate of evaporation of small drops of liquid in a gaseous atmosphere. The simplifying assumptions usually made are examined somewhat in detail, equations are derived which permit correction for most of these assumptions, and in a few cases the corrections are compared with experimental evidence.

The theory presented here is based mainly on the details of the processes of mass transfer during the evaporation of liquid droplets and partially on the processes of heat transfer.

J. Kaye, USA

568. G. Scatchard, L. F. Epstein, James Warburton, Jr., and P. J. Cody, "Thermodynamic properties—saturated liquid and vapor of ammonia-water mixtures," *Refrig. Engng.*, May 1947, vol. 52, pp. 413–419, 446, 448, 450, 452.

This article provides new values for the saturation pressure, vapor constitution, and enthalpy, entropy, free energy, and availability of both phases in the binary system NH_3–H_2O. The thermodynamic network is based on the following assumptions:

1 Amagat's law of additive volumes applies for both phases.
2 Compressibility for liquids is zero.
3 $pv/RT = [1 + f(T)\cdot p]$ for pure vapors.

The new values are probably the most accurate currently available. However, unexplained differences of as much as 45 psi exist between the authors' saturation pressures and those of Wilson, which are given in the International Critical Tables and in the Chemical Engineers' Handbook.

Equations [13], [24], and [26] of the text seem to be incorrectly stated. The reviewer has received private assurance that the erroneous form of Equation [26] was not used in the computations.

H. G. Elrod, Jr., USA

Heat Transfer

(See also Revs. 482, 493, 545, 556, 567)

569. H. S. Gardner and Irving Siller, "Shell-side coefficients of heat transfer in a baffled heat exchanger," *Trans. Amer. Soc. mech. Engrs.*, Aug. 1947, vol. 69, no. 6, pp. 687–694.

Experimentally determined shell-side convective heat-transfer coefficients for water and three oils, flowing in a half-moon baffled shell-and-tube heat exchanger, are presented, and are shown to be considerably lower for cooling than for heating conditions. An empirical viscosity-ratio correction is proposed to account for this variation. R. C. Martinelli, USA

570. M. F. Valerino and S. J. Kaufman, "Cylinder-temperature and cooling-air-pressure instrumentation for air-cooled-engine cooling investigations," *Nat. adv. Comm. Aero. Tech. Note No. 1509*, Jan. 1948, pp. 1–47.

A description is presented of the types and the locations of pressure tubes and thermocouples satisfactorily used by the National Advisory Committee for Aeronautics in multicylinder-engine cooling investigations. The advantages and disadvantages of the various types of pressure tubes and thermocouples are discussed with regard to reliability, durability, and ease of installation.

A. D. Kafadar, USA

571. G. V. Parmelee and R. G. Huebscher, "Heat transfer by forced convection along a smooth flat surface," *Heat. Pip. Air Condit.*, Aug. 1947, vol. 19, pp. 115–118.

Experimental data are presented for the case of forced convection from a horizontal surface to air flowing at velocities from 3 to 43 fps. The length of the constant temperature surface varied from 1.19 to 3.75 ft. Velocity distributions in the boundary layer were measured at several points along the plate.

Myron Tribus, USA

572. George Green, "Solutions of problems relating to media in contact by the method of wave-trains," *Phil. Mag. London*, Feb. 1947, vol. 38, pp. 97–115.

The solution of the Fourier-Poisson (heat conduction) equation in adjacent, concentric, circular cylindrical media is discussed by introducing the concept of "wave-trains." The boundary conditions at the mutual boundaries are formulated in terms of reflection and transmission coefficients, and solutions are given which automatically satisfy these conditions.

No particular advantage of this approach over more usual approaches is demonstrated. The reviewer feels that the term "wave-train" should not be applied to solutions of a parabolic equation. John W. Miles, USA

573. R. C. Martinelli, "Heat transfer to molten metals," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, pp. 947–959.

This paper contains more information than the title indicates, as it gives an extensive and advanced derivation and discussion of the equations concerning the "analogy between heat and momentum transfer" which was discussed earlier by the author and Boelter, after the original suggestion by von Kármán.

Analytical expressions for the temperature distribution in fluids flowing turbulently inside a circular duct or between two flat plates are first derived, and are used to determine relations between the common heat-transfer dimensionless groups, the Nusselt, Prandtl and Reynolds moduli. The "analogy" is said to exist in that the mechanism by which fluid turbulence determines the distribution of velocity or momentum also determines the temperature distributions in a flowing fluid. Experimental measurements of velocity distribution in the laminar sublayer, buffer layer and turbulent core of the flowing fluid are then used to determine the temperature distribution in the corresponding layers.

The paper presents the first complete derivation of these important heat-transfer equations from fundamental momentum and heat-balance relations. It shows that the equations for flow along flat plates are also useful for flow inside a circular duct. It presents an extension of the earlier work, to fluids of very low values of Prandtl modulus such as mercury, molten metals, etc. This is done by including a molecular heat-conduction term in parallel with the turbulent convective term in the turbulent core region of the flow. A tentative analysis of early data taken by Cope on roughened pipes is also given.

Valuable data and results are given in the form of graphs such as the temperature and velocity distributions in fluids covering a range of Prandtl and Reynolds moduli, the ratio of eddy diffusivity for momentum to kinematic viscosity in the three regions of the flow, and a comparison of results with "simpler analogy attempts" and with empirical equations.

The author has noted the following errors: On the basis of Equation [14] the limiting temperature distribution for zero Prandtl modulus, shown dotted in Figs. 3 and 4, should be parabolic and independent of the Reynolds modulus. The dotted portions of the curves in Figs. 6 and 9 should therefore be somewhat higher, and values of Nusselt modulus in Figs. 7 and 10 for the lower magnitudes of Prandtl and Reynolds moduli will be changed by 5 per cent or so. Earl H. Morrin, USA

574. Assène Datzeff, "On the cooling of a nonhomogeneous body (Sur le refroidissement d'un corps non homogène)," *C. R. Acad. Sci. URSS*, Jan. 20, 1947, vol. 55, no. 2, pp. 111–114.

Two homogeneous thin bars are in contact, one extending to $+\infty$, the other to $-\infty$. At zero time, the temperature distributions of both bars are known. The author demonstrates that at any later time the temperature at any point can be expressed by an integral equation in terms of the initial temperature distributions and the properties of the bars. A special form of the solution is required to predict interface temperatures.

Myron Tribus, USA

575. Z. F. Chukhanov, "Interior vs. exterior problem in heat exchange" (in English), *C. R. Acad. Sci. URSS*, Feb. 28, 1947, vol. 55, no. 6, pp. 497–500.

The author considers the problem of the correlation of drag incurred in flow over a flat plate and between two flat plates, by using a value of Reynolds modulus based upon the average velocity of flow in the boundary layer.

To correlate the data on heat transfer to materials of low Prandtl modulus, the author adds together the equation for laminar and turbulent flow of fluids. This correlates well the data of Strykovich for heat transfer to mercury. It is essentially similar to the method presented by Martinelli ["Heat transfer to molten metals," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, p. 947] in a recent paper, except that the Martinelli paper revises the turbulent-flow equations to add the molecular-conduction mode. (See Rev. 573.)

The author concludes that flow between parallel plates yields higher values of Nusselt modulus than does flow in round tubes, for a given power consumption. The reviewer had some difficulty in following this paper because of a lack of clear definitions of symbols, etc. Myron Tribus, USA